Name ____________________

MATH 121 Calculus and Analytic Geometry I Spring 2004 Project #5

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics*. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

The project is due in class on Wednesday, May 5. Your score on this project will count as extra credit.

Consider the function $f(x) = e^{-x^2}$ for the interval $[0, 1]$. We are interested in approximating the value of the definite integral $\int_0^1 e^{-x^2}\,dx$. You can think of this as giving the area of the region between the graph of f, the x-axis, and the vertical lines $x = 0$ and $x = 1$. You will get approximations by computing sums of rectangle areas. Let n be the number of subintervals used for a sum. Let L_n be the sum constructing using left endpoints and R_n be the sum constructing using right endpoints.

1. First, convince yourself that this function is decreasing on the interval $[0, 1]$ both by looking at a graph and by analyzing the first derivative. Thus, a sum L_n using left endpoints will overestimate the area and a sum R_n using right endpoints will underestimate the area.

2. The exact area A is bounded between the left endpoint sum and the right endpoint sum:

$$R_n \leq A \leq L_n.$$

   Convince yourself that the difference between the left endpoint sum and the right endpoint sum gives an upper bound on the possible error in either approximation. That is, the quantity $L_n - R_n$ is an upper bound on the difference between the exact area A and either L_n or R_n.

3. Aproximate the exact area to within a tolerance of ± 0.02. Start by computing the left and right endpoints sums with $n = 2$. Double the number of subintervals until the difference $L_n - R_n$ is less than the tolerance. You might find it useful to use a spreadsheet or the programable features of a calculator.

4. Look for a pattern in the difference $L_n - R_n$ as n is doubled. Use this pattern to predict how many subintervals are needed to approximate the exact area to within a tolerance of ± 0.0000001. Note that you do not need to compute the sum for this value of n, just determine how many subintervals are needed.

Note: The function $f(x) = e^{-x^2}$ and the value of integrals such as $\int_a^b e^{-x^2}\,dx$ play a important role in many probability problems.