

Instructions

You should submit a carefully written report addressing the problems given below. You are encouraged to discuss ideas with others for this project. If you do work with others, you must still write your report independently.

Use the writing conventions given in *Some notes on writing in mathematics*. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. All graphs should be done carefully on graph paper or using appropriate technology.

The project is due in class on Friday, February 27.

For each of the functions given below, do the following:

- (a) Plot the graph of the function around $(0, f(0)) = (0, 0)$. Explain (with words and/or plots) what is seen when you zoom in on $(0, f(0)) = (0, 0)$. Do you see a straight line? If so, what is the slope?
- (b) Analyze $f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$. (Give a direct analysis of this limit of a difference quotient. Do not use the rules of differentiation to compute this derivative.)
- (c) Compare your results for (a) and (b). Reconcile any differences.

1. $f(x) = x^{1/3}$

2. $f(x) = \tan x$

3. $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$