Name _____

MATH 352 Complex Analysis Spring 2003 Final Exam

Instructions: Do your own work. You may consult class notes, the course text, or other books. Give a reference if you use some source other than class notes or the course text. Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). If you are not confident in some result, you will receive more credit if you make a note of this and comment on where you might be going wrong or on alternate approaches you might try. The exam is due Friday, May 16 at 10:00 am.

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1. Prove the identity $\tan\left[\frac{1}{i}\log\left(\frac{1+iz}{1-iz}\right)^{1/2}\right] = z$.

2. Evaluate $\int_C (z^2+1)^{1/2}\,dz$ where C is the circle of radius $\frac{1}{2}$ centered at the origin and $w^{1/2}$ is defined using the branch of the square root function with $-\pi < \arg w < \pi$.

3. Evaluate $\int_0^\infty \frac{x^{1/4}}{1+x^5}\,dx$.

4. Prove any two of the following three claims. Submit attempts for only two.

   (a) If a function f is analytic on a domain A, then the function g defined by $g(z) = \overline{[f(\bar{z})]}^2$ is analytic on the domain $\bar{A} = \{z \mid \bar{z} \in A\}$.

   (b) There is no function f that has both of the following properties: f is analytic everywhere except the origin and f has derivative $f'(z) = \frac{1}{z}$ for $z \neq 0$.

   (c) If $P(z)$ is a polynomial and C is a simple closed curve with no roots of $P(z)$ on C, then $\frac{1}{2\pi i}\oint_C \frac{P'(z)}{P(z)}\,dz$ equals the number of roots of $P(z)$ inside C counting multiplicities.

5. Suppose the Math Club publishes a monthly newsletter for math and science majors. Write an article that describes complex analysis for this newsletter. Consider your target audience to be math and science majors who have completed the calculus sequence and linear algebra but who have *not* taken complex analysis. Your goal is to give those students guidance in making an informed decision about taking a course in complex analysis. Focus on aspects of complex analysis that you think are important or interesting. You do not need to summarize every idea that we have covered this semester.

   I will evaluate your article using the criteria described in the rubric on the reverse.