# More problems on conservative fields and potential functions

1. For each of the following, determine if the given vector field is conservative. If so, find a potential function for the field and use the potential to evaluate $\int_C \vec{F} \cdot d\vec{s}$ for a curve $C$ that starts at the origin and ends at $(3, 2, -7)$.

   (a) $\vec{F}(x, y, z) = \langle y, z\cos(yz) + x, y\cos(yz)\rangle$

   (b) $\vec{F}(x, y, z) = yz\hat{\imath} + xz\hat{\jmath}$

2. Consider the magnetic field due to a straight wire carrying a current $I$. In cylindrical coordinates with the $z$-axis running along the wire, this is

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}.$$

   (a) Show that the curl of this magnetic field is $\vec{0}$ for $\vec{r} \neq 0$.

   (b) Show that $-\frac{\mu_0 I}{2\pi}\arctan\left(\frac{x}{y}\right)$ is a potential function for this field.

   (c) Evaluate $\oint_C \vec{B} \cdot d\vec{s}$ where $C$ is the circle of radius 1 in the $xy$-plane centered at $(x, y) = (0, 3)$ by parametrizing the curve.

   (d) Explain why the result in (c) is exactly the result guaranteed by the Fundamental Theorem of Calculus for Line Integrals.

   (e) Evaluate $\oint_C \vec{B} \cdot d\vec{s}$ where $C$ is the circle of radius 1 in the $xy$-plane centered at $(x, y) = (0, 0)$ by parametrizing the curve.

   (f) Carefully explain why the result in part (e) does or does not contradict the Fundamental Theorem of Calculus for Line Integrals.