The gradient in various coordinate systems

For each of the functions in Problems 1-4, compute the gradient $\vec{\nabla} f(\vec{r})$ in terms of polar coordinates (r, θ) and polar basis vectors $\{\hat{r}, \hat{\theta}\}$ and make a plot of this vector field.

1. $f(r, \theta) = \dfrac{1}{r}$
2. $f(r, \theta) = \theta$
3. $f(r, \theta) = \cos\theta$
4. $f(r, \theta) = \dfrac{\cos\theta}{r}$

For Problems 5-6, consider a function $f : \mathbb{R}^3 \to \mathbb{R}$.

5. Find an expressions for $d\vec{r}$ and $\vec{\nabla} f(\vec{r})$ in terms of cartesian coordinates (x, y, z) and basis vectors $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$.
6. Find an expressions for $d\vec{r}$ and $\vec{\nabla} f(\vec{r})$ in terms of cylindrical coordinates (r, θ, z) and basis vectors $\{\hat{r}, \hat{\theta}, \hat{k}\}$.
7. Find an expressions for $d\vec{r}$ and $\vec{\nabla} f(\vec{r})$ in terms of spherical coordinates (ρ, ϕ, θ) and basis vectors $\{\hat{\rho}, \hat{\phi}, \hat{\theta}\}$.
8. Use your results from Problem 7 to compute the gradient of the function $f(\rho, \phi, \theta) = \rho^n$.