# Some more problems on divergence and curl

1. Prove each of the following identities.

   (a) $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = 0$ for a vector field $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$

   (b) $\vec{\nabla} \cdot (\vec{\nabla} f \times \vec{\nabla} g) = 0$ for a pair of functions $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^3 \to \mathbb{R}$

2. Consider a vector field of the form $\vec{F}(\vec{r}) = r^n \hat{r}$ in a $k$-dimensional universe. For example, the $k = 2$ universe is the plane for which $\vec{r} = \langle x, y \rangle$ and the $k = 3$ universe is ordinary space for which $\vec{r} = \langle x, y, z \rangle$. We can also consider universes of dimension $k > 3$.

   (a) Find an expression for $\vec{\nabla} \cdot \vec{r}$ in dimension $k$.

   (b) Find a general expression for $\vec{\nabla} \cdot \vec{F}(\vec{r}) = \vec{\nabla} \cdot (r^n \hat{r})$. Note that this expression will involve both the dimension $k$ and the power $n$. Hint: Use a product rule, your result for (a), and a result about gradients from earlier in the semester.

   (c) Consider static electric fields $\vec{E}$ in "flatland," that is , the $k = 2$ universe. Suppose that $\vec{\nabla} \cdot \vec{E} = 0$ for the electric field due to a point charge (at any point other than the position of the point charge). From this, deduce the form of Coulomb's law in the $k = 2$ universe. Compare this to Coulomb's law in the $k = 3$ universe.