Notes on notation for line integrals

In class, we defined the line integral of the vector field $\vec{F}$ for the curve C as

$$\int\limits_C \vec{F} \cdot d\vec{s} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt.$$

The text uses an alternate notation for the line integral. Here's the connection: Write the vector field $\vec{F}$ in terms of components as $\vec{F} = u\,\hat{\imath} + v\,\hat{\jmath} + w\,\hat{k}$ and write the vector $d\vec{s}$ in terms of components as $d\vec{s} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$. Here, think of dx as a small displacement parallel to the x-axis, dy as a small displacement parallel to the y-axis, and dz as a small displacement parallel to the z-axis. With these component expressions, we can write out the dot product as

$$\vec{F} \cdot d\vec{s} = u\,dx + v\,dy + w\,dz.$$

Using this, the notation for line integral is

$$\int\limits_C \vec{F} \cdot d\vec{s} = \int\limits_C u\,dx + v\,dy + w\,dz.$$

The text favors the expression on the right side and I generally use the expression on the left side.

Most of the problems are given using the notation on right side. For example, Problem 3 involves the line integral

$$\int_C (-y\,dx + x\,dy).$$

From this, you can read off that the vector field is $\vec{F}(x, y) = \langle -y, x\rangle$ (or $-y\,\hat{\imath} + x\,\hat{\jmath}$ if you prefer).