Physics 122MA/Math 221PH
Spring 2002 Project 8 Name ______________________________

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is required that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, April 30.

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1. Consider a (positive) point charge q_0 at the origin.

   (a) Write an expression for the electric field at a point (x, y, z). Express your answer as a vector that depends only on q_0, the coordinates x, y, and z, and constants.

   (b) Use your answer to (a) to compute $\vec{\nabla} \cdot \vec{E}$ at all points (x, y, z) excluding the point $(0, 0, 0)$.

   (c) Is it possible to find $\vec{\nabla} \cdot \vec{E}$ at the origin? Explain.

   (d) Show that your results in both (b) and (c) are consistent with Equation (27.22), which states $\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0$.

2. Consider a vector field of the form $\vec{F}(\vec{r}) = r^n \hat{r}$ in a k-dimensional universe. For example, the $k = 2$ universe is the plane for which $\vec{r} = \langle x, y \rangle$ and the $k = 3$ universe is ordinary space for which $\vec{r} = \langle x, y, z \rangle$. We can also consider universes of dimension $k > 3$.

   (a) Find a general expression for div $\vec{F}(\vec{r}) = \vec{\nabla} \cdot (r^n \hat{r})$. Note that this expression will involve both the dimension k and the power n.

   (b) Consider static electric fields $\vec{E}$ in "flatland," that is , the $k = 2$ universe. Suppose that $\vec{\nabla} \cdot \vec{E} = 0$ for the electric field due to a point charge (at any point other than the position of the point charge). From this, deduce the form of Coulomb's law in the $k = 2$ universe. Compare this to Coulomb's law in the $k = 3$ universe.