Physics 122MA/Math 221PH
Spring 2002 Project 7 Name ____________________

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is required that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, April 23.

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1. A long straight cylindrical wire of radius R carries a current density that is not uniform but varies according to the function

$$\vec{J} = I_0 \frac{r^2}{R^4} \widehat{k}$$

where I_0 is a constant, r is the distance from the center of the wire, and the z-axis is taken to run along the central axis of the cylinder.

   (a) Find the magnetic field everywhere, inside and outside the wire. Your answers should be functions of the variable r and the parameters I_0 and R.

   (b) Use the current density and magnetic field from Part (a) to show that $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ for $r < R$ and $\vec{\nabla} \times \vec{B} = 0$ for $r > R$.

2. Prove each of the following identities.

   (a) $\operatorname{div} \operatorname{curl} \vec{F} = 0$ for a vector field $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$

   (b) $\operatorname{div}(\vec{\nabla} f \times \vec{\nabla} g) = 0$ for a pair of functions $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^3 \to \mathbb{R}$