Physics 122MA/Math 221PH
Spring 2002 Project 6 Name ____________________

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is required that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, April 9.

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1. Derive Gauss's law for gravitation. *Hint*: Consider a point particle of mass m and find the gravitational field $\vec{g}$ in the space around the particle. Then compute the flux of the gravitational field

$$\Phi_g = \oiint \vec{g} \cdot \hat{n} \, dA$$

through a Gaussian sphere, and argue that your result is true in general for a Gaussian surface enclosing a mass.

2. Consider the vector field

$$\vec{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle .$$

(a) Show that the function $f(x, y) = -\arctan(x/y)$ satisfies

$$\frac{\partial f}{\partial x} = \frac{-y}{x^2 + y^2} \qquad \text{and} \qquad \frac{\partial f}{\partial y} = \frac{x}{x^2 + y^2}.$$

(b) Use the definition given by formula 22.5 on page 742 of the text to compute $\int_C \vec{F} \cdot d\vec{s}$ where C is the circle $(x-2)^2 + (y-3)^2 = 1$ parametrized by $\vec{r}(t) = \langle 2 + \cos(t), 3 + \sin(t) \rangle$ for $t \in [0, 2\pi]$.

(c) Explain why the result in (b) is exactly the result guaranteed by the Fundamental Theorem of Calculus for Line Integrals.

(d) Use the definition given by formula 22.5 on page 742 of the text to compute $\int_C \vec{F} \cdot d\vec{s}$ where C is the circle $x^2 + y^2 = 1$ parametrized by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $t \in [0, 2\pi]$.

(e) Carefully explain why the result in part (d) does or does not contradict the Fundamental Theorem of Calculus for Line Integrals.