Physics 122MA/Math 221PH
Spring 2002 Project 4 Name ______________________________

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is required that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, February 26.

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1. Consider a thin charged rod that lies from $x = 1$ m to $x = 2$ m on the x-axis. the rod has a uniform charge density $\lambda = 1.0 \times 10^{-6}$ C/m. We are interested in computing the value of the electric field at the point (3m, 3m).

   (a) Do a numerical estimate of the electric field by breaking the rod into 20 equal pieces, computing the approximate electric field at (3m, 3m) due to each piece, and then summing. A spreadsheet would be extremely helpful.

   (b) Express the electric field at (3m, 3m) in terms of definite integrals by taking the limit as the number of intervals becomes infinite.

   (c) Evaluate the definite integrals you found in part (b). You may use a computer to perform the evaluation. If you do so, please provide a hard copy of the program's output and specify which software you used.

   (d) Now change to a *non-uniform charge density* $\lambda(x) = (x^2 + 1.0) \times 10^{-6}$ with outputs in C/m for inputs in m. Repeat part (a) for this situation.

2. The unit basis vectors $\hat{\imath}$ and $\hat{\jmath}$ are defined with respect to a cartesian coordinate system. At a point in the plane with cartesian coordinates (x_0, y_0), the vector $\hat{\imath}$ has a direction tangent to the line given by $y = y_0$. Of the two possible directions, $\hat{\imath}$ points in the direction of increasing x. In a similar fashion, the vector $\hat{\jmath}$ has a direction tangent to the line given by $x = x_0$. Of the two possible directions, $\hat{\jmath}$ points in the direction of increasing y.

   Now consider two new vectors $\hat{r}$ and $\hat{\theta}$ defined to be the unit basis vectors for a polar coordinate system in the following manner. At a point in the plane with polar coordinates (r_0, θ_0), the vector $\hat{r}$ is defined as a unit vector in the direction tangent to the ray defined by $\theta = \theta_0$. Of the two possible directions, $\hat{r}$ points in the direction of increasing r. The vector $\hat{\theta}$ is defined as a unit vector in the direction tangent to the circle defined by $r = r_0$. Of the two possible directions, $\hat{\theta}$ points in the direction of increasing θ.

   (a) Make a figure showing both the cartesian unit basis vectors and the polar unit basis vectors at several points in the plane. In what ways is the set of polar unit basis vectors $\{\hat{r}, \hat{\theta}\}$ similar to the set of cartesian unit basis vectors $\{\hat{\imath}, \hat{\jmath}\}$ and in what ways is it different?

   (b) For a point in the plane with polar coordinates (r, θ), express the vectors $\hat{r}$ and $\hat{\theta}$ in terms of the vectors $\hat{\imath}$ and $\hat{\jmath}$. In other words, find the cartesian components of the vectors $\hat{r}$ and $\hat{\theta}$.

(c) Make a plot of each of the following vector fields.

1. $\vec{F}(r, \theta) = \hat{\theta}$
2. $\vec{F}(r, \theta) = \dfrac{1}{r}\hat{\theta}$
3. $\vec{F}(r, \theta) = \dfrac{1}{r^2}\hat{r}$

(d) Use your results for (b) and the transformation equations between polar and cartesian coordinates to express the vector field $\vec{F}(r, \theta) = \frac{1}{r}\hat{\theta}$ entirely in terms of cartesian coordinates (i.e., in terms of x, y, $\hat{\imath}$, and $\hat{\jmath}$).