

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is required that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, February 5.

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1. In Section 15.5, we showed that any spherically symmetric distribution of mass with total mass  $M$  has the same gravitational effect on a particle outside the sphere as a single particle of mass  $M$  located at the center of the sphere. In general, this does *not* hold for any other mass distribution, even one with a lot of symmetry. Consider, for example, a one-dimensional distribution of mass along the  $x$ -axis from  $x = 0$  to  $x = L$ . Assume a uniform mass density and total mass  $M$ . Now place a particle of mass  $m$  on the  $x$ -axis at  $x = d$  with  $d > L$ .
  - (a) Construct a definite integral that will compute the magnitude of the force on  $m$  due to its gravitational attraction to  $M$ .
  - (b) Evaluate the definite integral from (a).
  - (c) Show that your result in (b) is not the same as you would obtain by replacing the linear distribution with a single particle of mass  $M$  at the distribution's center of mass ( $x = L/2$ ).
  - (d) Analyze what happens to your results in (b) and (c) in the case  $d \gg L$ .
  
2. A line in the plane can be described by a linear equation in two variables of the form  $Ax + By + C = 0$ . Alternatively, a line in the plane can be described parametrically by a vector function of the form  $\vec{r}(t) = \vec{r}_0 + t\vec{d}$  where  $\vec{r}_0$  is the position vector for a point on the line and the direction vector  $\vec{d}$  is parallel to the line.

A plane in space can be described by a linear equation in three variables of the form  $Ax + By + Cz + D = 0$ . Come up with a way to describe a plane in space parametrically using a vector function. Hint: Think about using two parameters.