Name ______________________________

MATH 221A | Multivariate Calculus | Spring 2002 | Project #8

**Instructions**

You are encouraged to work with others on this project. As with all writing you should work out the details in draft form before writing a final solution. You should write your solution in paragraph form using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. You should do your own writing of the solution even if you have worked out the details with other people. All graphs should be done carefully on graph paper or using appropriate technology. The project is due at the beginning of class on Friday, May 3.

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The unit basis vectors $\hat{\imath}$ and $\hat{\jmath}$ are defined with respect to a cartesian coordinate system. At a point in the plane with cartesian coordinates $(x_0, y_0)$, the vector $\hat{\imath}$ has a direction tangent to the line given by $y = y_0$. Of the two possible directions, $\hat{\imath}$ points in the direction of increasing $x$. In a similar fashion, the vector $\hat{\jmath}$ has a direction tangent to the line given by $x = x_0$. Of the two possible directions, $\hat{\jmath}$ points in the direction of increasing $y$.

Now consider two new vectors $\hat{r}$ and $\hat{\theta}$ defined to be the unit basis vectors for a polar coordinate system in the following manner. At a point in the plane with polar coordinates $(r_0, \theta_0)$, the vector $\hat{r}$ is defined as a unit vector in the direction tangent to the ray defined by $\theta = \theta_0$. Of the two possible directions, $\hat{r}$ points in the direction of increasing $r$. The vector $\hat{\theta}$ is defined as a unit vector in the direction tangent to the circle defined by $r = r_0$. Of the two possible directions, $\hat{\theta}$ points in the direction of increasing $\theta$.

A word on notation: Some people use the notation $\hat{u}_r$ and $\hat{u}_\theta$ in place of $\hat{r}$ and $\hat{\theta}$ to denote the unit basis vectors for polar coordinates. The analogs for the cartesian unit basis vectors would be $\hat{u}_x$ and $\hat{u}_y$ in place of $\hat{\imath}$ and $\hat{\jmath}$. Another alternative for the cartesian unit basis vectors would be $\hat{x}$ and $\hat{y}$. This most closely resembles the notation $\hat{r}$ and $\hat{\theta}$ we are using for the polar unit basis vectors.

1. Make a figure showing both the cartesian unit basis vectors and the polar unit basis vectors at several points in the plane. In what ways is the set of polar unit basis vectors $\{\hat{r}, \hat{\theta}\}$ similar to the set of cartesian unit basis vectors $\{\hat{\imath}, \hat{\jmath}\}$ and in what ways is it different?

2. For a point in the plane with polar coordinates $(r, \theta)$, express the vectors $\hat{r}$ and $\hat{\theta}$ in terms of the vectors $\hat{\imath}$ and $\hat{\jmath}$. In other words, find the cartesian components of the vectors $\hat{r}$ and $\hat{\theta}$.

3. Make a plot of each of the following vector fields.
   (a) $\vec{F}(r, \theta) = \hat{\theta}$
   (b) $\vec{F}(r, \theta) = \dfrac{1}{r}\hat{\theta}$
   (c) $\vec{F}(r, \theta) = \dfrac{1}{r^2}\hat{r}$

4. Use your results for Problem 3 and the transformation equations between polar and cartesian coordinates to express the vector field $\vec{F}(r, \theta) = \frac{1}{r}\hat{\theta}$ entirely in terms of cartesian coordinates (i.e., in terms of $x$, $y$, $\hat{\imath}$, and $\hat{\jmath}$).