Name ______________________________

MATH 221A Multivariate Calculus Spring 2002 Project #3

Instructions

You are encouraged to work with others on this project. As with all writing you should work out the details in draft form before writing a final solution. You should write your solution in paragraph form using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. You should do your own writing of the solution even if you have worked out the details with other people. All graphs should be done carefully on graph paper or using appropriate technology. The project is due at the beginning of class on Monday, February 25.

The sphere of radius 1 centered at the origin can be described by the parametric equation

$$\vec{r} = \langle \sin(s)\cos(t), \sin(s)\sin(t), \cos(s) \rangle$$

where the parameter s has values in the interval $[-\pi/2, \pi/2]$ and the parameter t has values in the interval $[-\pi, \pi]$.

1. Argue that this parametric equation does describe the sphere of radius 1 centered at the origin by doing each of the following.
   (a) Show that for any choice of s and t, the vector $\vec{r}$ is a position vector for a point on the sphere.
   (b) Show that for any point (x, y, z) on the sphere, there is a choice of s and t for which the vector $\vec{r}$ gives that point.
2. (a) Consider fixing a value of s and letting t vary through the values in $[-\pi, \pi]$. The vector $\vec{r}$ traces out a curve on the sphere. Describe such a curve in the language we use to describe a globe (north pole, south pole, equator, longitude, latitude, etc.).
   (b) Consider fixing a value of t and letting s vary through the values in $[-\pi/2, \pi/2]$. The vector $\vec{r}$ traces out a curve on the sphere. Describe such a curve in the language we use to describe a globe (north pole, south pole, equator, longitude, latitude, etc.).
3. Consider the point on the sphere given by $s = \pi/6$ and $t = \pi/4$. Compute a vector normal (i.e., perpendicular) to the sphere by taking the cross product of two vectors, one tangent to the curve of constant s through the point and the other tangent to the curve of constant t through the point.