Name ______________________________

MATH 221A   Multivariate Calculus   Spring 2002   Project #1

Instructions

You are encouraged to work with others on this project. As with all writing you should work out the details in draft form before writing a final solution. You should write your solution in paragraph form using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. You should include enough detail so that a reader can follow your reasoning and reconstruct your work. You should not show every algebraic or arithmetic step. You should do your own writing of the solution even if you have worked out the details with other people. All graphs should be done carefully on graph paper or using appropriate technology. The project is due at the beginning of class on Tuesday, February 5.

Physicists and chemists model a crystal as a three-dimensional lattice of points with a single atom centered at each point. To describe a lattice, pick a set of three vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ which have a right-hand orientation. The lattice can then be thought of as the set of all points with position vectors of the form $n_1\vec{v}_1 + n_2\vec{v}_2 + n_3\vec{v}_3$ where n_1, n_2, and n_3 are integers.

For a set of lattice vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$, the *reciprocal lattice vectors* are defined as

$$\vec{k}_1 = \frac{\vec{v}_2 \times \vec{v}_3}{\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)}, \qquad \vec{k}_2 = \frac{\vec{v}_3 \times \vec{v}_1}{\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)}, \qquad \text{and} \qquad \vec{k}_3 = \frac{\vec{v}_1 \times \vec{v}_2}{\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)}.$$

These reciprocal lattice vectors are useful in analyzing the physical properties of crystals. Here we will look only at some of the mathematical results which follow from the definitions given above. The identities you will prove give some justification to the name “reciprocal” lattice vectors.

1. Show that $\vec{k}_1 \cdot \vec{v}_1 = 1$.

2. Show that $\vec{k}_1 \cdot (\vec{k}_2 \times \vec{k}_3) = \dfrac{1}{\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)}$.

   Hint: Use the “bac-cab” identity: $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$.