Name ______________________________

MATH 302 Partial Differential Equations Fall 2011 Exam #7

Instructions: Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). All plots should be carefully drawn either by hand or printed from technology. If you want to include a visualization that cannot be printed (such as an animation), include it as an attachment in an email with "Math 302 Exam 7" as the subject line.

The exam is due on Friday, December 16 by 2 pm (which is the end of the scheduled final exam period for this course).

Pick *two* of the following three problems to submit. The two problems will be equally weighted in your overall score.

1. You want to model how your guitar will sound if you submerge it in a vat of honey. Honey resistance will be important so you decide to include a damping term in your model. A simple model is to assume a damping force that is proportional to velocity. Including a term of this form in the wave equation gives us the PDE
$$u_{tt} = c^2 u_{xx} - \beta u_t.$$
   (a) Set up an initial-boundary value problem for this PDE that models vibrations on a string of length l with both ends held fixed in which the string is displaced from equilibrium and then released from rest.

   (b) Find a solution to the IBVP you set up in (a) for the case $\beta < \frac{2\pi c}{l}$. (This condition guarantees underdamping for all modes.)

   (c) Illustrate your solution from (b) using specific parameter values (string length l, wave speed c, and damping coefficient β) and a specific initial displacement of your own choice other than the trivial case of zero initial displacement.

2. Set up and solve the general Dirichlet boundary-value problem for Laplace's equation on an annular region (that is, the region between two concentric circles). Illustrate a specific non-trivial solution using your own choice of boundary conditions.

3. Consider vibrations of a membrane in the shape of a half-disk with edges held fixed in a plane. Find the five smallest normal mode frequencies. Illustrate each of the corresponding modes.