

Instructions: Do your own work. You may consult your class notes and the course text. Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). All plots should be carefully drawn either by hand or printed from technology. If you want to include a visualization that cannot be printed (such as an animation), include it as an attachment in an email with "Math 302 Exam 5" as the subject line.

The exam is due at the beginning of class on Monday, November 7.

One way to generate an orthogonal set is use the *Gram-Schmidt process* starting with a linearly independent set that is not necessarily orthogonal.

- (a) Start with the set $\{1, x, x^2, x^3, x^4, x^5, \dots\}$ in $L^2[-1, 1]$. Use the Gram-Schmidt process to compute at least the first six elements of an orthogonal set.
- (b) Compute at least the first six terms in the expansion of $f(x) = \sin(\pi x)$ in terms of the orthogonal set from (a).
- (c) Develop conjectures about the convergence of the orthogonal expansion from (b). Give evidence or argument to support each conjecture you make.