A Sturm-Liouville problem

Consider the Sturm-Liouville problem

$$\begin{aligned} y''(x) &= -\lambda y(x) \qquad \text{for } 0 < x < l \\ y(0) &= 0 \\ y(l) + y'(l) &= 0 \end{aligned}$$

1. Use a Rayleigh quotient (or energy) argument to bound the set of eigenvalues. Hint: Multiply both sides of the ODE by y and then integrate both sides from $x = 0$ to $x = l$.

2. Find a "characteristic equation" for the eigenvalues.

3. Use your equation from 2 to argue that there are countably many eigenvalues λ_n that can be ordered as $\lambda_1 < \lambda_2 < \ldots$ with $\lambda_n \to \infty$ as $n \to \infty$.
Hint: Think graphically.

4. Estimate λ_1, λ_2, and λ_3.

5. Come up with a formula that gives approximate values of λ_n for large values of n.

6. Find the eigenfunctions.