Boundary value problems for Laplace's equation

In each of the following, you are given a physical region in which heat can flow and a description of how the boundary of that region is controlled. Write down a complete boundary value problem for the steady-state heat energy density distribution (or equivalently, temperature distribution). Use your intuition to sketch level curves (i.e., isotherms) for the steady-state distribution.

1. Heat flows in a rectangular region. Going around the edges in a particular direction:
   - the first edge is held at 0°
   - the second edge is perfectly insulated
   - the third edge is perfectly insulated
   - the fourth edge is held at 100°
2. Heat flows in a rectangular region. Going around the edges in a particular direction:
   - the first edge is held at 0°
   - the second edge is perfectly insulated
   - the third edge is perfectly insulated
   - the fourth edge is open to the external environment which has a constant temperature
3. Heat flows in an equilateral triangular region. One edge is held at 0° while the other two edges are held at 100°.
4. Heat flows in a circular region. The temperature on one half of the boundary is held at 0° while the temperature on the other half is held at 100°.
5. Heat flows in a semi-circular region. The temperature on the diameter is held at 0°. The circular arc is perfectly insulated.