MATH 280   Multivariate Calculus   Spring 2011   Exam #3

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.   (100 points total)

1. Consider the function $f(x, y) = \dfrac{x^2}{x + y}$.

   (a) Find the equation of the tangent plane for the point $(x_0, y_0) = (1, 2)$.   (12 points)

   (b) A flea sits on the graph of the function over the point $(1, 2)$. The flea jumps up in the direction perpendicular to the tangent plane at this point. Find this direction. (Note: You can find any vector in the correct direction. You do not need to give the result as a unit vector.)   (4 points)

2. Consider the function $f(x, y, z) = \dfrac{x + 2y}{z^2}$.

   (a) Find the linearization for this function based at the point $(3, 1, 2)$.   (10 points)

   (b) Use the linearization to approximate $f(3.2, 0.9, 2.1)$.   (5 points)

   (c) Give an upper bound on the error in the approximation from (b). For this you can use the fact that the absolute values of all second derivatives of f are bounded above by 48 for the region with $2 \leq x \leq 4$, $0 \leq y \leq 2$, and $1 \leq z \leq 3$.   (5 points)

3. To determine the (constant) velocity V of an object in motion, you can measure a distance D traveled, measure the corresponding amount of time T, and then compute $V = D/T$.

   (a) Find the linear relation among the differentials dV, dD, and dT.   (10 points)

   (b) Find a relation among (infinitesimal) percentage changes in V, D, and T. Express this relation both as a formula and in a sentence.   (6 points)

4. Explain the distinction between a *local maximum* and a *global maximum*.   (8 points)

There is more on the flip side.

5. Do either *one* of the following two problems. Circle the number of the problem you submit.

(20 points)

(A) Find all local extremes and saddle points for the function $f(x,y) = x^2 y - 6xy + y^2$.

(B) Find the global extremes for the function $f(x,y) = xye^{-y}$ on the region with $0 \le x \le 2$ and $0 \le y \le 2$.

6. Do either *one* of the following two problems. Circle the number of the problem you submit.

(20 points)

(A) Consider a bundle of two goods, say apples and bananas. Let a be the amount of apples in the bundle and b be the amount of bananas, both measured in pounds. Model the consumer utility for this bundle with the function

$$U = \left(\tfrac{1}{4}a^{1/2} + \tfrac{3}{4}b^{1/2}\right)^2.$$

Suppose that apples cost $2 per pound and bananas cost $6 per pound. Use the method of Lagrange multipliers to find the bundle that optimizes utility for a consumer who has $100 to spend on apples and bananas.

(B) A *Norman window* has the shape of a rectangle surmounted by a semi-circle as shown in the figure below. Use the method of Lagrange multipliers to find the dimensions of the Norman window that has minimum perimeter for a given area.