Name _____

MATH 280 Multivariate Calculus Spring 2011 Exam #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Consider the function $f(x, y) = \sqrt{16 - x^2 - y^2}$.
   (a) Determine the domain of this function. (4 points)
   (b) Determine the range of this function. (3 points)
   (c) Sketch representative level curves for this function. (6 points)
   (d) Sketch and/or describe the graph of this function. (4 points)

2. Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^3 y}{x^4 + 5y^4}$ does not exist. (8 points)

3. (a) For $f(x, y, z)$, state the **definition** of *partial derivative of f with respect to z.* (4 points)
   (b) For $f(x, y, z)$, state an **interpretation** of *partial derivative of f with respect to z.* (4 points)

4. Compute the second partial derivatives of $f(x, y) = y \sin(xy)$. (15 points)

5. The ideal gas law relates pressure p, volume V, temperature T, and number of gas particles n as $pV = nRT$ where $R = 0.082$ L·atm/(mol·K) is a constant. Suppose we are doing an experiment with $n = 1$ mol of gas held constant. During the experiment, there is a particular time at which the pressure has the value 1.4 atm and is changing at the rate of 0.3 atmospheres per hour while the volume has value 0.8 L and is changing at the rate of -0.1 liters per hour. For this particular time, what is the rate at which the temperature is changing with respect to time? (10 points)

6. Honey bees sense a specific chemical to find a certain flower species. Suppose that in a particular region of space, the concentration of this chemical varies from point to point according to the function $c(x, y, z) = x^2y^4 + 3z^2$. A bee hovers at the point $P(3, 2, 5)$. The bee wants to start following the path along which the chemical concentration increases most rapidly. In what direction should the bee go initially? Give your answer as a unit vector. (10 points)

There is more on the flip side.

7. Compute the rate of change in $f(x, y, z) = x^2yz^3$ at the point $P(2, 4, 1)$ in the direction of the point $Q(-3, 1, 5)$. (12 points)

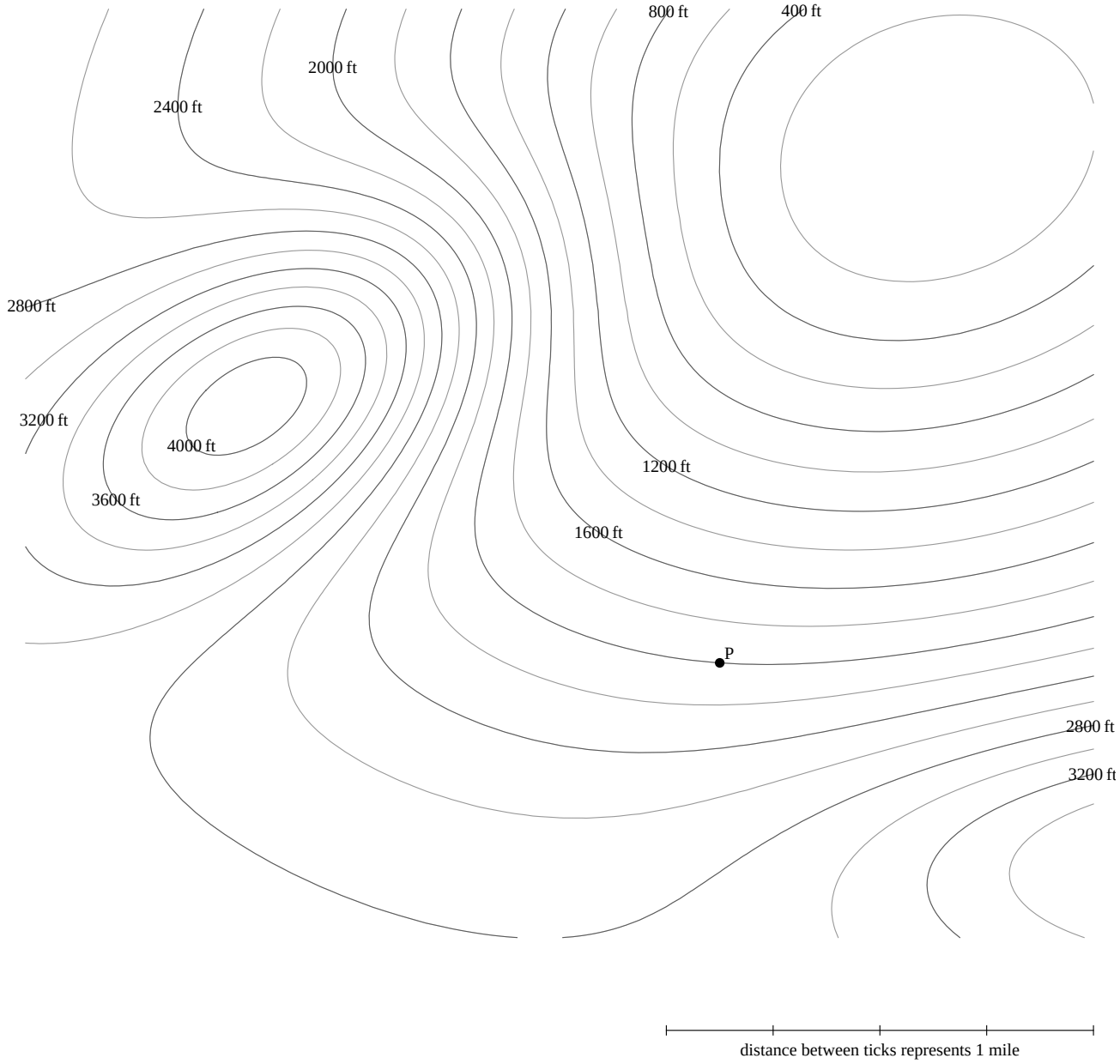
8. The accompanying plot shows level curves for elevation (measured in feet) of some landscape. Note that the distance scale for the map plane is in miles.

   (a) Estimate the gradient of the elevation at the point marked P. Show the direction of your estimate on the plot as an arrow at P and write down your estimate of the magnitude. (6 points)

   (b) Estimate the location of the largest elevation gradient vector. Indicate your estimated location on the plot with a point labeled A. What is special or interesting about this place in the landscape? (4 points)

   (c) Estimate the location of a point at which the elevation gradient vector is $\vec{0}$. Indicate your estimated location on the plot with a point labeled B. What is special or interesting about this place in the landscape? (4 points)

9. Prove that $\vec{\nabla}(k\,f) = k\vec{\nabla}f$ for any constant k and any differentiable function f. (6 points)



Plot for Problem 8

Level curves for elevation (in feet) over a planar region (with map distances in miles)