Name ____________________

MATH 280 Multivariate Calculus Spring 2011 Composite 2 of Exams #1 and #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. Draw any two vectors u and v that are not equal in magnitude and not perpendicular. Use your two vectors to draw (with reasonable accuracy) each of the following.

   (a) $2\vec{u} + \frac{1}{2}\vec{v}$ (b) $2\vec{u} - \frac{1}{2}\vec{v}$

2. Give a geometric argument for the fact that $\vec{u} + \vec{v} = \vec{v} + \vec{u}$. (8 points)

3. Consider the vectors $\vec{u} = 7\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{v} = 3\hat{\imath} + 4\hat{k}$.

   (a) Compute $3\vec{u} - 2\vec{v}$.

   (b) Find the projection of $\vec{v}$ in the direction of $\vec{u}$.

4. Consider two vectors $\vec{u}$ and $\vec{v}$ for which $\|\vec{u}\| = 4$, $\|\vec{v}\| = 3$, and $\vec{u} \cdot \vec{v} = -2$. Use the given information to compute $(\vec{u} - 2\vec{v}) \cdot (\vec{u} + 5\vec{v})$.

5. A flea sits on the plane given by the equation $4x + 5y + 10z = 16$. The flea launches itself into the air with an initial velocity $\vec{v} = 0.2\hat{\imath} - 0.4\hat{\jmath} + 0.5\hat{k}$ (in units of inches per second). Find the angle between this initial velocity and a direction perpendicular to the plane.

6. Do either one of the following two problems. Circle the letter for the problem you submit.

   (A) Use vectors to show that the diagonals of a parallelogram bisect each other.

   (B) Show that it is *not* possible to have vectors $\vec{u}$ and $\vec{v}$ with $\|\vec{u}\| = 5$, $\|\vec{v}\| = 2$, and $\vec{u} \cdot \vec{v} = -12$.

7. Honey bees sense a specific chemical to find a certain flower species. Suppose that in a particular region of space, the concentration of this chemical varies from point to point according to the function $c(x, y, z) = x^2y^4 + 3z^2$. A bee hovers at the point $P(3, 2, 5)$. The bee wants to start following the path along which the chemical concentration increases most rapidly. In what direction should the bee go initially? Give your answer as a unit vector.

8. Compute the rate of change in $f(x, y, z) = x^2yz^3$ at the point $P(2, 4, 1)$ in the direction of the point $Q(-3, 1, 5)$.

There is more on the flip side.

9. The accompanying plot shows level curves for elevation (measured in feet) of some landscape. Note that the distance scale for the map plane is in miles.

   (a) Estimate the gradient of the elevation at the point marked P. Show the direction of your estimate on the plot as an arrow at P and write down your estimate of the magnitude.

   (b) Estimate the location of the largest elevation gradient vector. Indicate your estimated location on the plot with a point labeled A. What is special or interesting about this place in the landscape?

   (c) Estimate the location of a point at which the elevation gradient vector is $\vec{0}$. Indicate your estimated location on the plot with a point labeled B. What is special or interesting about this place in the landscape?

Below are some problems from older exams that deal with vector-valued functions.

1. The position of a fly buzzing around the room is given by

$$\vec{r}(t) = \left(\tfrac{1}{3}t^3 - 9t + 2\right)\hat{\imath} + \left(\tfrac{1}{2}t^2 - 3t - 4\right)\hat{\jmath} + \left(5t + t^2\right)\hat{k}$$

   (in inches) where $\hat{\imath}$ points due east, $\hat{\jmath}$ points due south, and $\hat{k}$ points up.

   (a) Where is the fly at $t = 2$?

   (b) Compute the velocity function for the fly.

   (c) Find a point at which the fly is heading straight up.

2. A piece of wire is wrapped around the elliptic cylinder given by the equation $\frac{1}{9}x^2 + y^2 = 1$ to form an elliptic helix. The wire wraps counterclockwise from the viewpoint of looking down the z-axis. For each wrap around the cylinder, the helix rises 3 units in the z-direction. Find a parametrization for four wraps of this elliptic helix starting from the point $(3, 0, 0)$. Give a range of values for your parameter.

3. Think of a vector output function $\vec{r} : \mathbb{R} \to \mathbb{R}^3$ as tracing out a curve in space.

   (a) State a *geometric* interpretation for an output of the derivative $\vec{r}\,'(t)$ in relation to the curve traced out by $\vec{r}$.

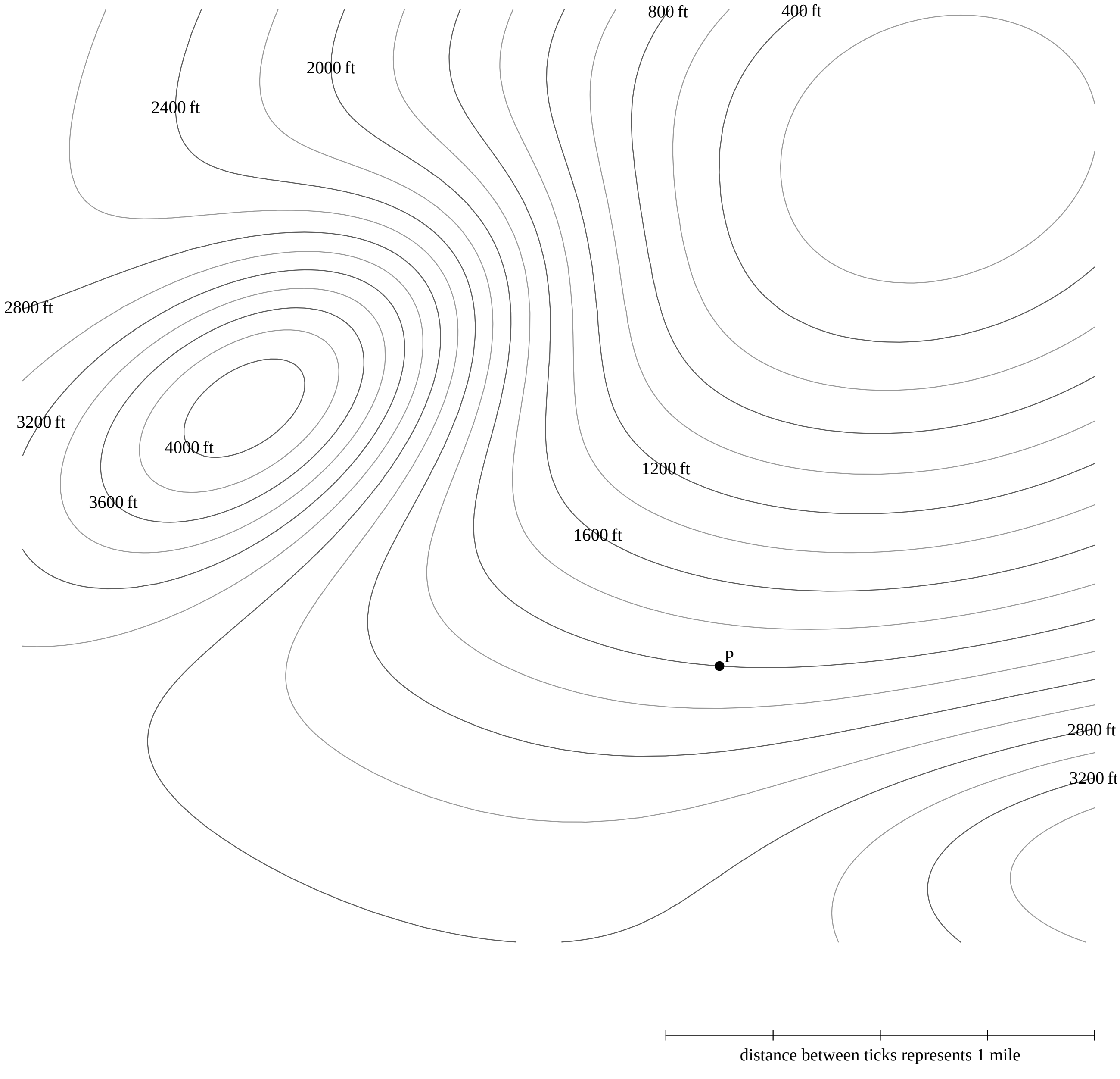
   (b) Justify the interpretation you give in (a) starting from the definition of the derivative $\vec{r}\,'$.

4. Consider the curve traced out by $\vec{r}(t) = 5t^2\,\hat{\imath} + 6t\,\hat{\jmath} + (1 + t^3)\,\hat{k}$ and the plane given by $x - 2y + z = 5$.

   (a) Show that the curve intersects the plane for $t = 2$.

   (b) Find a vector tangent to the curve at the point of intersection corresponding to $t = 2$.

   (c) Find the angle between the tangent vector from (b) and a normal vector for the plane.



Plot for Problem 9

Level curves for elevation (in feet) over a planar region (with map distances in miles)