Name ________________________________

MATH 280 Multivariate Calculus Fall 2010 Exam #5

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

A. Pay attention to the statement of each problem. If a problem asks you to "Set up a definite/iterated integral or integrals..." you do not need to evaluate the integral(s).

B. For full credit, each definite integral you set up should be expressed entirely in terms of one variable and each iterated integral you set up should be expressed entirely in terms of one pair of variables.

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1. Provide a brief answer to each of the following questions.

   (a) What, at a fundamental level, is a line integral for a vector field? (4 points)

   (b) What, at a fundamental level, is a surface integral for a scalar field? (4 points)

2. Find a vector that is perpendicular to both of the vectors $\vec{u} = 3\,\hat{\imath} - 2\,\hat{\jmath} + \hat{k}$ and $\vec{v} = -\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$. (6 points)

3. Set up a definite integral or integrals equal to $\displaystyle\int_C f\,ds$ where $f(x, y) = 5xy$ and C is the piece of the cubic curve $y = x^3$ for $x = 0$ to $x = 2$. (16 points)

4. Set up a definite integral or integrals to compute the length of a helix that wraps 10 times around the elliptical cylinder $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ extending from $z = 0$ to $z = 5$.
   Hint: An ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is traced out once by $x = a\cos\theta$ and $y = b\sin\theta$ for θ from 0 to 2π. (16 points)

5. Charge is distributed on the surface of a right circular cone of radius R and height H so that the area charge density is proportional to the distance from the central axis of the cone. Compute the total charge on the cone in terms of R, H, and the maximum charge density σ_0. (16 points)

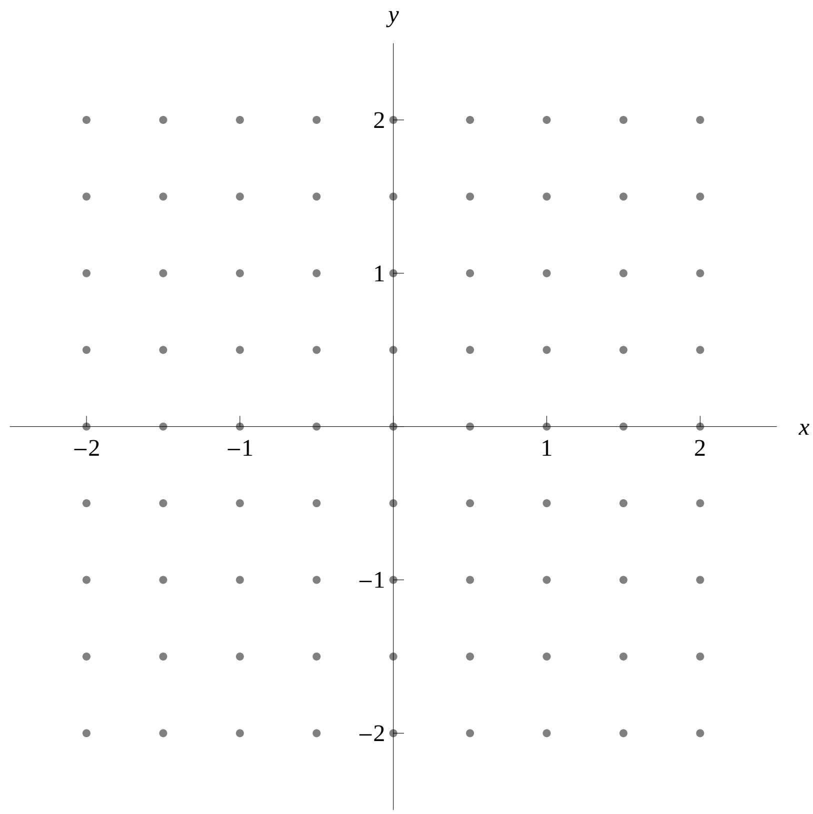
There is more on the flip side.

6. (a) Use the grid below to sketch the vector field $\vec{F} = y\,\hat{\imath} + 2\,\hat{\jmath}$ for the region with $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Plot an output for each of the points provided on the grid. (4 points)

   (b) Sketch the curve $y = x^2 - 2$ for $x = -2$ to $x = 2$ on the same plot below. (2 points)

   (c) Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}$ as given in (a) and C as the curve in (b). (12 points)

   (d) Use your plot as a check on the reasonableness of your result in (c). Explain your thinking in reaching a conclusion. (4 points)

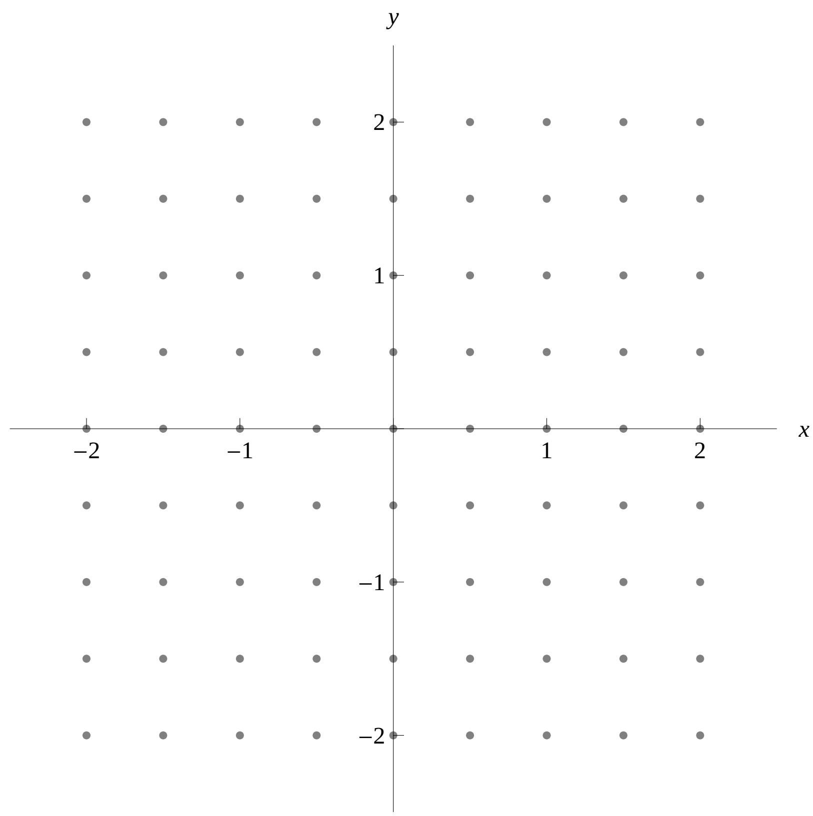


7. Consider the vector field $\vec{F} = \left(1 + \dfrac{1}{y}\right)\hat{\imath} + \left(z^2 - \dfrac{x}{y^2}\right)\hat{\jmath} + 2yz\,\hat{k}$ for $y > 0$.

   (a) Use the component test to show that $\vec{F}$ is conservative. (4 points)

   (b) Find a potential function for $\vec{F}$. (8 points)

   (c) Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where C is a curve that starts at $(1, 1, 0)$ and ends at $(0, 2, 3)$. (4 points)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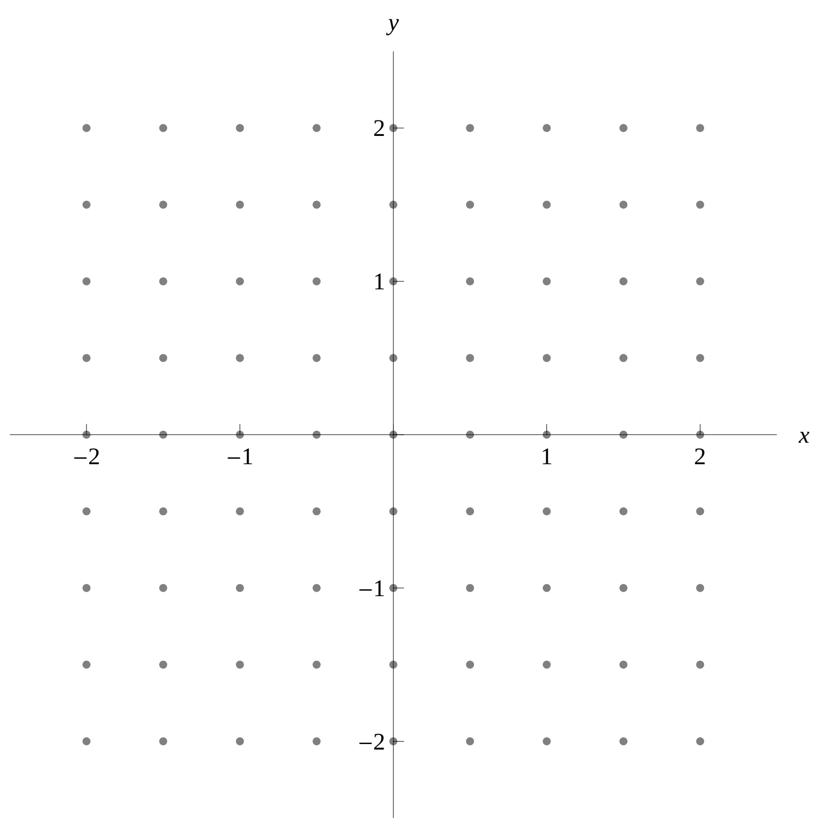
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