Name _____

MATH 280 Multivariate Calculus Fall 2010 Exam #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

For this exam, you should not use symbolic features on your calculator if they are available on your model.

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1. Consider the function $z = f(x, y) = \sqrt{1 - xy}$.
   (a) Determine the domain of this function. (4 points)
   (b) Determine the range of this function. (2 points)
   (c) Sketch level curves of this function for $z = 0$, $1/2$, 1, $3/2$, and 2. (6 points)
   (d) Use pictures and/or words to describe the graph of this function. (4 points)

2. Show that $\lim_{(x,y)\to(0,0)} \dfrac{y \sin x}{x^2 + y^2}$ does not exist. (8 points)

3. Analyze continuity of the function $f(x, y) = \dfrac{1}{x^2 + y^2 - 4}$. That is, determine all values of (x, y) for which f is continuous and describe any discontinuities of f. (8 points)

4. State the definition of the partial derivative $\dfrac{\partial H}{\partial q}$ for a function $H(p, q)$. (Give a definition, not an interpretation such as rate of change or slope. Note that the definition involves a difference quotient.) (6 points)

5. Consider the function $f(x, y) = e^{x^2 y}$.
   (a) Compute the first partial derivatives of f. (8 points)
   (b) Compute the second partial derivatives of f. (8 points)

There is more on the flip side.

6. Consider a consumer who can purchase different amounts of three commodities: apples, bananas, and cherries. Let a, b, and c be the amount purchased of each (measured in pounds). A simple model used by economists assigns a utility U (in units we'll call *utils*) to each bundle (a, b, c) the consumer can purchase according to the formula

$$U = k\, a^{1/2} b^{1/6} c^{1/3}$$

where $k = 1$ util/lb (to keep units consistent). Compute the rate of change in utility with respect to change in the amount of bananas purchased for the bundle (5 lbs, 2 lbs, 4 lbs).

(8 points)

7. The temperature on a tabletop is given by $T(x, y) = k\, xy$ where $k = 15°\text{C/m}^2$ and (x, y) are cartesian coordinates measured in meters from one corner of the table. A bug passes through the point $(0.5, 0.2)$ m moving with velocity $0.03\hat{\imath} - 0.05\hat{\jmath}$ m/s. Find the rate of change in temperature with respect to time for this bug as it passes through $(0.5, 0.2)$ m.

(10 points)

8. Consider the function $f(x, y, z) = x^2 yz^3$ and the point $P(-1, 5, 2)$ in the domain of f.

   (a) Compute the gradient of f at P. (5 points)

   (b) Compute the greatest rate of change in f at P. (4 points)

   (c) Describe the nature of level surfaces through and near P in a zoomed in view. (3 points)

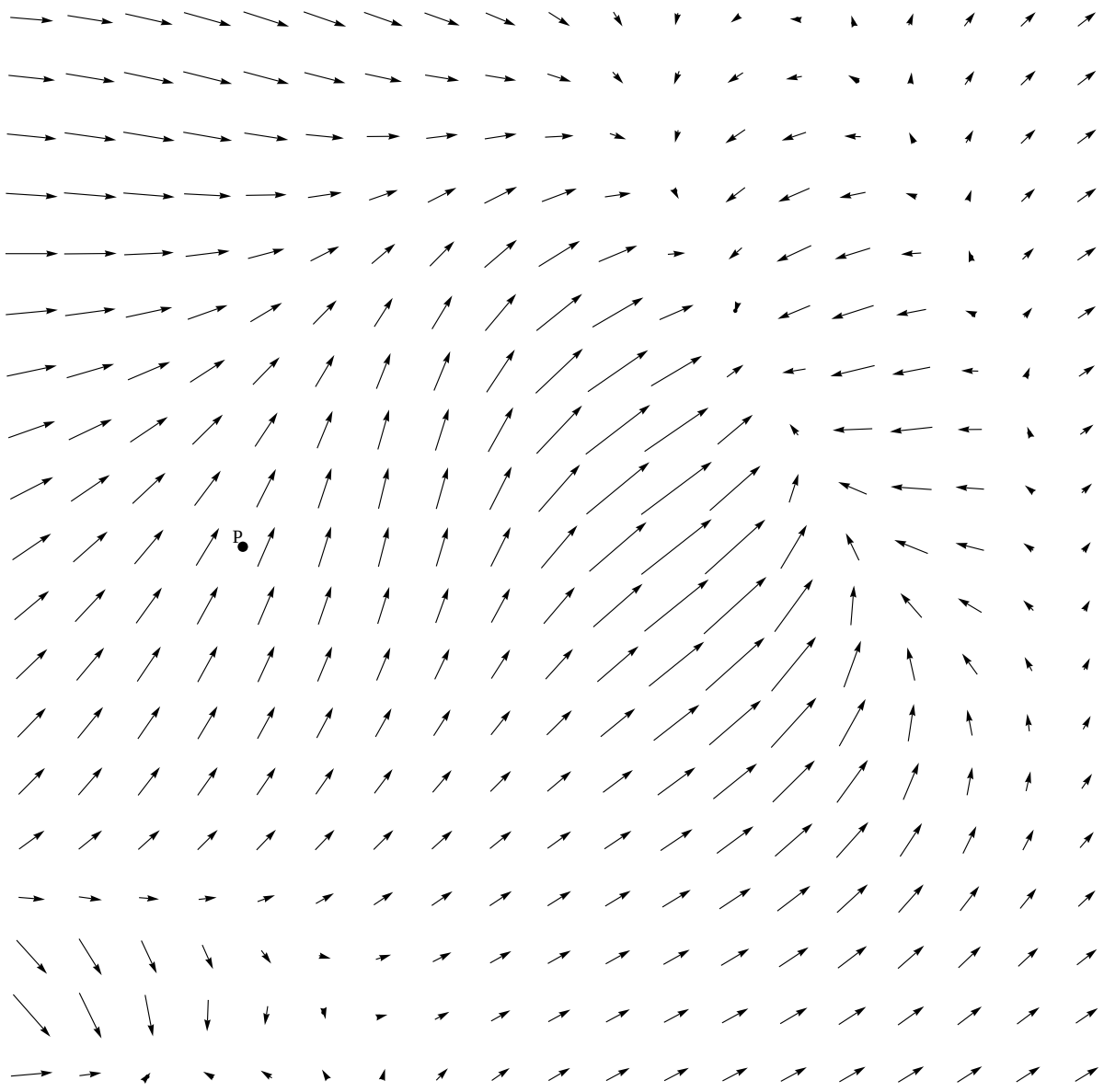
   (d) Compute the rate of change in f at P in the direction of $Q(2, 1, 4)$. (6 points)

9. The accompanying plot show gradient vectors for temperature (in °C) considered as a function of position on a plane.

   (a) Suppose a heat-loving ant starts at the point P labeled in the plot. The ant can sense the temperatures in all directions a very small distance aways from its position. The ant moves along a path that is tangent to the direction of most rapid increase in temperature at each point. On the plot, sketch an estimate of the path this ant will take. (5 points)

   (b) Another ant starts at the point P. This ant likes the temperature at P. On the plot, sketch an estimate of a path this ant can take to move around while staying at the same temperature. (5 points)

   Note: Clearly label the two paths on your plot with (a) and (b) as appropriate.



Plot for Problem 9