

Part A Instructions: Do the work for these problems on separate paper. Show enough detail for me to assess how you arrive at your conclusions. Clearly box or circle the final answer for each computational problem. Do reasonable simplification. Each problem has a value of 5 points. *You may not use the symbolic features of a calculator such as those on a TI-89 for this part of the exam.*

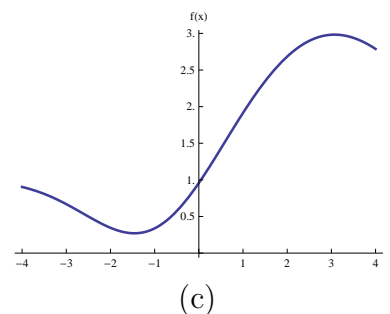
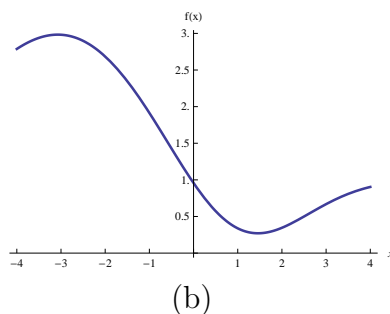
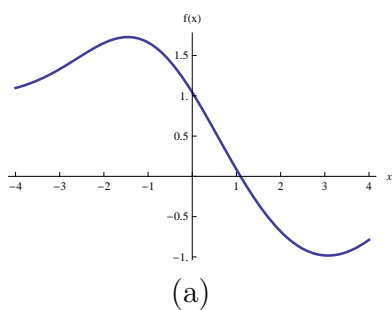
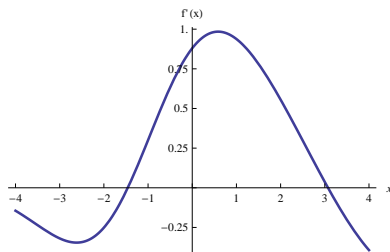
1. State the definition of *derivative*.
2. State the definition of *definite integral*.
3. State the Second Fundamental Theorem of Calculus. Include the hypotheses and the conclusion.

4. Evaluate the limit $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$.

5. Evaluate the limit $\lim_{x \rightarrow 0} \frac{x \cos(x)}{\sin(2x)}$.

6. Determine if $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 3 \\ 12 & \text{if } x = 3 \\ 16 - 2x & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

7. Use the definition (as limit of a difference quotient) to compute the derivative of $f(x) = x^2$.
8. The plot below shows the graph of a derivative function f' . Below it are three other plots. Determine which of the three shows the graph of the function f .



9. Compute the derivative of $f(x) = 4x^3 - 5x^2 + 2x + 1$.
10. Compute the derivative of $f(x) = \sqrt{\sin x}$.
11. Compute the derivative of $f(z) = e^{3z} \cos(4z)$.
12. Compute the derivative of $g(t) = \frac{t^3}{t^2 + 1}$.
13. Compute the derivative of $f(x) = \tan^{-1}(x^3)$.
14. Compute the derivative of $f(x) = Ae^{-kx}$ where A and k are constants.
15. Compute the *second* derivative of $f(x) = x \ln x$.
16. Find all values of x for which $f(x) = x^3 - 9x^2$ has a horizontal tangent line.
17. The position of an object moving along a straight line is given by $s(t) = 4 \sin(\pi t)$. Find the velocity of the object for time $t = 2$.
18. For a particular amount of an ideal gas held at a particular constant temperature, the pressure p and volume V are related by $pV = 10$. Find the rate of change in pressure with respect to volume.
19. Show that $x = 3$ is a critical point for the function $f(x) = x^2 + \frac{54}{x}$ and classify this critical point as a local minimum, a local maximum, or neither.
20. Use the Mean Value Theorem to show that the function $f(x) = 10\sqrt{x+3}$ has a tangent line with slope 2 for some x between $x = 1$ and $x = 6$.
21. Find all antiderivatives of the function $f(x) = 2x^6 - 5x$.
22. Find all antiderivatives of the function $f(x) = \frac{6}{x} + \sin(x)$.
23. An object moving along a straight line has velocity given by $v(t) = 6t^2$ and is at position $s = 0$ for $t = 1$. Find the position function for the object.
24. Evaluate the definite integral $\int_{-2}^3 (x^2 + 4) dx$.
25. Find the area of the region between the graph of $\sin(x)$ and the x -axis between $x = 0$ and $x = \pi$.

Part B Instructions: Do any three of the following five problems. Do the work for these problems on separate paper. Show enough detail for me to assess how you arrive at your conclusions. On this page, circle each of the problem numbers for the three problems you are submitting. Each problem has a value of 15 points.

Note: For these problems you can “buy” geometry formulas for 1 point each.

- I. Show that the point $(1, 3)$ is on the curve given by $y^3 - 7x^3y - x = 5$ and find the slope of the tangent line to the curve at this point.
- II. Consider the function $f(x) = \frac{x}{x^2 + a}$ where a is a positive constant. Use calculus techniques to do the following.
- (a) Show that the function has a local maximum at $x = \sqrt{a}$.
 - (b) Show that the function has an inflection point at $x = \sqrt{3a}$.
- III. You are watching your friend run a sprint at a track meet. You have a seat that is 10 meters away from the track. The start line is to your left and the finish line is to your right so your head rotates as you watch the race. Your friend is moving at 6 meters per second when she passes directly in front of you. How fast is your head rotating at that instant?
- IV. In your job as an engineer, you are given the task of designing a cylindrical steel tank that holds 300 cubic meters. The tank is to have a base but no top. Find the dimensions that minimize the amount of steel used.
- V. Use the definition of definite integral to compute the value of $\int_1^3 x^2 dx$. That is, evaluate this using the limit of a Riemann sum rather than using the Second Fundamental Theorem of Calculus.