For this project, you have a choice of three options. Your project report should allow readers to follow your analysis and understand your insights. Your report should also be self-contained so you should start by setting up some context with your own version of the problem statement or background. Refer to previous handouts on writing in mathematics for general directions.

You should focus on doing your own mathematical analysis rather than looking for resources in which similar problems or contexts are analyzed. If you do use ideas or material from resources other than our textbook, you must give a proper citation. You should not discuss details of this project with other students who may have done a similar project in previous semesters.

The project is due on Tuesday, November 29.

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**Option 1**

For this option, your objective is to analyze the problem of maximizing the volume of a (right circular) cylinder inscribed in an ellipsoid. Aim for a general analysis, although you might find it useful to start with special cases (such as a sphere).

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**Option 2**

**Background** In Option 2 for Project #1, we looked at *utility functions*. We think of a consumer purchasing a *bundle* consisting of certain amounts of different commodities. *Utility* is an abstract measure of the value of each bundle to the consumer. For simplicity, we limit attention here to having a commodity bundle with just two types of thing. Let $x$ and $y$ denote the amount of each thing in a bundle. For convenience, we can think of $x$ and $y$ as measuring weight in pounds (lb). We'll use the made-up unit *util* for the unit for utility $U$. One model for utility is the *constant elasticity of substitution (CES)* utility function given by

$$U = \left[ax^b + (1-a)y^b\right]^{1/b}.$$

For $a$, we use $0 \le a \le 1$; for $b$, we use $b \le 1$.

**Objective** For this option, your objective is to analyze the problem of maximizing the CES utility with respect to $x$ and $y$ for a fixed budget $B$. As part of this, you should explore how any solution you find depends on the various parameters in the problem.

## Option 3

**Background:** A hydrogen atom consists of one proton and one electron. A *free* hydrogen atom is one that experiences no external forces. In a free hydrogen atom, the electron can be in one of infinitely many discrete states. These states are labeled by three integers, usually denoted $n$, $l$, and $m$. For each state, there is an *electron location probability density* that gives the volume probability density for the location of the electron as a function of position (measured with respect to the proton).

The $n = 3$, $l = 2$, $m = 0$ state of a free hydrogen atom has an electron location probability density given by

$$\delta(\rho, \phi, \theta) = \frac{1}{39366\pi}\rho^4 e^{-2\rho/3}(3\cos^2\phi - 1)^2$$

where $(\rho, \phi, \theta)$ are spherical coordinates as we use them in class. The origin of the coordinate system is the location of the proton. The radial coordinate $\rho$ is measured in units of *Bohr radii* where the Bohr radius is equal to about $5.3 \times 10^{-11}$ meters. (So, for example, $\rho = 2$ means a radial distance of 2 Bohr radii.)

**Objective:** For this option, your objective is to get insight on this probablility density function. As part of this, you should compute some total probabilities for specific regions of space.