MATH 280   Multivariate Calculus   Spring 2011   Exam #4

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

   This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.   (100 points total)

A. Throughout this exam, x, y, and z refer to cartesian coordinates; r and θ refer to polar coordinates (so r, θ and z refer to cylindrical coordinates); and ρ, ϕ, and θ refer to spherical coordinates as we have defined these in class.

B. Pay attention to the statement of each problem. If a problem asks you to "Set up an iterated integral or integrals..." you do not need to evaluate the integral(s).

C. For full credit, each iterated integral you set up should be expressed entirely in terms of one coordinate system.

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1. Consider a situation in which a total charge Q is to be computed as $Q = \iint\limits_{R} \sigma \, dA$, using the notation we have been using in class.

   (a) In a brief phrase or sentence, state what $\iint$ represents.   (2 points)

   (b) In a brief phrase or sentence, state what R represents.   (2 points)

   (c) In a brief phrase or sentence, state what σ represents.   (2 points)

   (d) In a brief phrase or sentence, state what dA represents.   (2 points)

   (e) In a sentence or two, describe how we typically compute a value for $\iint\limits_{R} \sigma \, dA$. (4 points)

2. Give a geometric justification for the form of the area element in polar coordinates. That is, explain how to get the formula $dA = r \, dr \, d\theta$. Include a relevant picture.   (10 points)

3. A geometric argument for the volume element in spherical coordinates can be made by considering the spherical "box" corresponding to going from ρ to $\rho + d\rho$, ϕ to $\phi + d\phi$, and θ to $\theta + d\theta$. Express the volume element in spherical coordinates in a form that makes clear how it can be seen as a product of three relevant lengths.   (6 points)

There is more on the flip side.

4. Compute the value of the double integral $\iint\limits_R f \, dA$ where $f(x, y) = y$ and R is the region in the first quadrant bounded by the lines $y = 0$, $x - y = 0$ and $2x + y = 6$. (16 points)

5. Set up an iterated integral or integrals equal to the total charge on a square of side length L on which charge is distributed with area charge density proportional to the distance from the center reaching a maximum σ_0 at each of the four corners. (14 points)

6. Do either *one* of the following two problems. Circle the number of the problem you submit. (14 points)

   (A) Set up an interated integral or integrals equal to the volume of the region bounded by the plane $z = 0$, the plane $x + z = 5$, and the cylinder $x^2 + y^2 = 9$.

   (B) Set up an interated integral or integrals equal to the triple integral $\iiint\limits_D f \, dV$ where $f(x, y, z) = 3z$ and D is the half of the solid sphere of radius 5 centered at the origin with $z \geq 0$.

7. Do either *one* of the following two problems. Circle the number of the problem you submit. (14 points)

   (A) Consider a solid cone of height H and radius R having non-uniform composition with volume mass density proportional to the distance from the central axis, reaching a maximum of δ_0 on the surface. Compute the total mass.

   (B) Consider a solid sphere of radius R having non-uniform composition with volume mass density proportional to the the distance from the surface, reaching a maximum δ_0 at the center. Compute the total mass.

8. Do either *one* of the following two problems. Circle the number of the problem you submit. (14 points)

   (A) Set up an iterated integral or integrals equal to the area of one petal of the polar curve $r = \sin(4\theta)$.

   (B) Set up a definite integral or integrals equal to the total length of the polar curve $r = \theta$ for $0 \leq \theta \leq 2\pi$.