Name ______________________________

MATH 280 Multivariate Calculus Spring 2011 Composite of Exams #1 and #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Explain why $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ is the equation of the sphere of radius r centered at the point (x_0, y_0, z_0). (9 points)

2. Consider the surface given by the quadratic equation $4x^2 - y^2 - 9z^2 = 1$.
   (a) Sketch the cross-sections of the surface for $x = -1$, $x = 0$, and $x = 1$. (9 points)
   (b) Sketch the cross-section of the surface for $y = 0$. (3 points)
   (c) Sketch the cross-section of the surface for $z = 0$. (3 points)
   (d) Use pictures and/or words to describe the surface given by this quadratic equation. (6 points)

3. Find the standard form for the equation of the plane containing the points $(5, -2, 1)$, $(5, 2, 7)$, and $(10, 2, 2)$. (12 points)

4. Consider the function $f(x, y) = \sqrt{16 - x^2 - y^2}$.
   (a) Determine the domain of this function. (4 points)
   (b) Determine the range of this function. (3 points)
   (c) Sketch representative level curves for this function. (6 points)
   (d) Sketch and/or describe the graph of this function. (4 points)

5. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^4 + 5y^4}$ does not exist. (8 points)

There is more on the flip side.

6. (a) For $f(x, y, z)$, state the **definition** of *partial derivative of* f *with respect to* z. (4 points)

   (b) For $f(x, y, z)$, state an **interpretation** of *partial derivative of* f *with respect to* z. (4 points)

7. Compute the second partial derivatives of $f(x, y) = y \sin(xy)$. (15 points)

8. The ideal gas law relates pressure p, volume V, temperature T, and number of gas particles n as $pV = nRT$ where $R = 0.082$ L·atm/(mol·K) is a constant. Suppose we are doing an experiment with $n = 1$ mol of gas held constant. During the experiment, there is a particular time at which the pressure has the value 1.4 atm and is changing at the rate of 0.3 atmospheres per hour while the volume has value 0.8 L and is changing at the rate of -0.1 liters per hour. For this particular time, what is the rate at which the temperature is changing with respect to time? (10 points)