Name ______________________________

MATH 280 Multivariate Calculus Spring 2011 Exam #1

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Consider the surface given by the quadratic equation $4x^2 - y^2 - 9z^2 = 1$.
   (a) Sketch the cross-sections of the surface for $x = -1$, $x = 0$, and $x = 1$. (9 points)
   (b) Sketch the cross-section of the surface for $y = 0$. (3 points)
   (c) Sketch the cross-section of the surface for $z = 0$. (3 points)
   (d) Use pictures and/or words to describe the surface given by this quadratic equation. (6 points)

2. Find the standard form for the equation of the plane containing the points $(5, -2, 1)$, $(5, 2, 7)$, and $(10, 2, 2)$. (12 points)

3. Find the x-slope, the y-slope and the z-intercept of the plane that contains the point $(-5, 1, 3)$ and is perpendicular to the vector $\langle 7, 3, 4 \rangle$. (10 points)

4. Draw any two vectors u and v that are not equal in magnitude and not perpendicular. Use your two vectors to draw (with reasonable accuracy) each of the following. (5 points each)
   (a) $2\vec{u} + \frac{1}{2}\vec{v}$ (b) $2\vec{u} - \frac{1}{2}\vec{v}$

5. Give a geometric argument for the fact that $\vec{u} + \vec{v} = \vec{v} + \vec{u}$. (8 points)

6. Consider the vectors $\vec{u} = 7\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{v} = 3\hat{\imath} + 4\hat{k}$.
   (a) Compute $3\vec{u} - 2\vec{v}$. (6 points)
   (b) Find the projection of $\vec{v}$ in the direction of $\vec{u}$. (6 points)

7. Consider two vectors $\vec{u}$ and $\vec{v}$ for which $\|\vec{u}\| = 4$, $\|\vec{v}\| = 3$, and $\vec{u} \cdot \vec{v} = -2$. Use the given information to compute $(\vec{u} - 2\vec{v}) \cdot (\vec{u} + 5\vec{v})$. (9 points)

There is more on the flip side.

8. A flea sits on the plane given by the equation $4x+5y+10z=16$. The flea launches itself into the air with an initial velocity $\vec{v}=0.2\hat{\imath}-0.4\hat{\jmath}+0.5\hat{k}$ (in units of inches per second). Find the angle between this initial velocity and a direction perpendicular to the plane.

(9 points)

9. Do any one of the following three problems. Circle the letter for the problem you submit.

(9 points)

(A) Use vectors to show that the diagonals of a parallelogram bisect each other.

(B) Explain why $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is the equation of the sphere of radius r centered at the point (x_0,y_0,z_0).

(C) Show that it is *not* possible to have vectors $\vec{u}$ and $\vec{v}$ with $\|\vec{u}\|=5$, $\|\vec{v}\|=2$, and $\vec{u}\cdot\vec{v}=-12$.