Name ______________________________

MATH 280 | Multivariate Calculus | Fall 2010 | Exam #3

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. Consider the temperature T as a function of position (x, y, z) in space. Suppose measurements are made to determine the values

$$T(1,4,2) = 27, \quad T_x(1,4,2) = 3.2, \quad T_y(1,4,2) = -1.3, \quad \text{and} \quad T_z(1,4,2) = 2.3.$$

   (a) Construct the linearization of T based at the point $(1, 4, 2)$. (8 points)

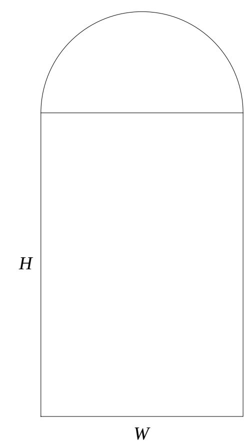
   (b) Use your linearization from (a) to estimate T at the point $(1.07, 4.15, 1.98)$. (6 points)

2. A *Norman window* has the shape of a rectangle surmounted by a semi-disc as shown in the figure. With W as the width of the rectangle and H as the height of the rectangle, the area of the window is given by

$$A = WH + \frac{1}{8}\pi W^2.$$

   (a) Compute the linear relation among the differentials dA, dW, and dH. (9 points)

   (b) Consider the values $W = 100$ and $H = 20$. For these values, use your relation from (a) to compare the effect on A of equal-size changes in W and H. (5 points)



3. Consider the function $f(x, y) = x\sin(x + y)$

   (a) Show that $(0, \pi)$ is a critical point of f.
   Hint: You can do this by setting up the relevant equations and then substituting in the given point to show it is a solution. (7 points)

   (b) Use the second-derivative test to classify $(0, \pi)$ as a local minimizer, a local maximizer, or neither. (7 points)

There is more on the flip side.

4. Consider the function $f(x, y) = xy(2 - y)$. For this function, the only critical points are $(0, 0)$ and $(0, 2)$. Find the global minimum and global maximum of this function for the rectangle $-1 \leq x \leq 4$, $-1 \leq y \leq 2$. (15 points)

5. Do either *one* of the following two problems. Circle the number of the problem you submit. (16 points)

   (A) In your job as a movie theater manager, you want to determine optimal prices for child tickets and adult tickets. Let p_1 be the price of a child ticket and p_2 be the price of an adult ticket (both measured in dollars). Also, let q_1 and q_2 be the quantity sold for each ticket type. You understand that the quantity sold will depend on the price. Based on some data, you decide to model the relationships between price and quantity sold using

   $$q_1 = \frac{1000}{p_1^2 p_2} \qquad \text{and} \qquad q_2 = \frac{2500}{p_2^2}.$$

   Your operating costs are \$3 each child ticket sold and \$4 for each adult ticket sold. Find the prices that maximize your profits.
   Hint: Profit is total revenue minus total cost.

   (B) Consider a (right circular) cylinder segment. Use the method of Lagrange multipliers to find the dimensions that give the maximum volume for a given surface area (including top and bottom).

6. Compute the value of the double integral $\iint_D f\, dA$ where $f(x, y) = 2xy + x^2$ and D is the region in the *second* quadrant of the xy-plane bounded by the curves $x = 0$, $y = 0$, and $y = 9 - x^2$. As a small part of this, point out where you use Fubini's Theorem.
   Note: You can stop when only arithmetic remains. (15 points)

7. Consider a thin rectangular plate of dimensions L by W. The materials composing the plate vary from point to point so that the area mass density is porportional to the square of the distance from the center of the plate, reaching a maximum of σ_0 at each of the four corners. Set up an iterated integral to compute the total mass of the plate.
   Note: You do *not* need to evaluate the iterated integral you set up. (12 points)