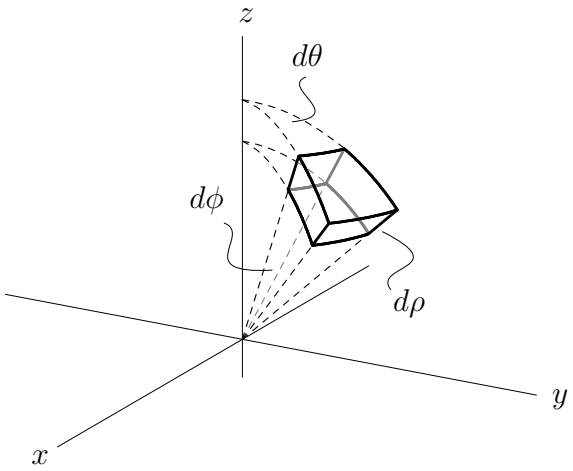
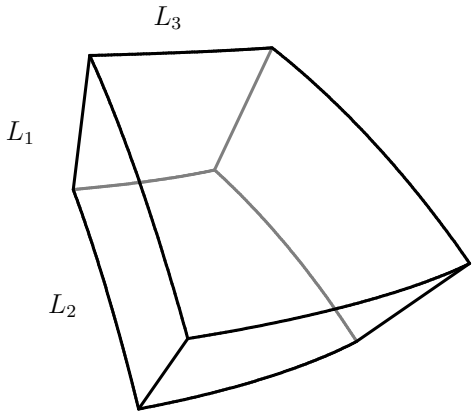
The volume element in spherical coordinates

The figure below on the left shows a generic spherical "box" defined as the points with spherical coordinates ranging in intervals of extent $d\rho$, $d\phi$, and $d\theta$. Let (ρ, ϕ, θ) be the spherical coordinates of some particular point in the box. The figure on the right shows a zoomed-in view of the box with the edge lengths labeled L_1, L_2, and L_3.





1. What is L_1 in terms of ρ, ϕ, θ, $d\rho$, $d\phi$, and $d\theta$ (as needed)?

2. The length L_2 is the length of an arc of a circle.

   (a) What is the radius of this circle?

   (b) What is the angle subtended by the arc?

   (c) What is L_2 in terms of ρ, ϕ, θ, $d\rho$, $d\phi$, and $d\theta$ (as needed)?

3. The length L_3 is also the length of an arc of a circle.

   (a) What is the radius of this circle?
   Hint: Draw a vertical cross-section of the sphere.

   (b) What is the angle subtended by the arc?

   (c) What is L_3 in terms of ρ, ϕ, θ, $d\rho$, $d\phi$, and $d\theta$ (as needed)?

4. Let dV be the volume of the box. What is dV in terms of ρ, ϕ, θ, $d\rho$, $d\phi$, and $d\theta$ (as needed)?