# Differentiability

Math 280

Fall 2011

# Idea of differentiability

For $f : \mathbb{R} \to \mathbb{R}$

- Idea: $f$ is differentiable at $x_0$ if zooming in on the graph at $(x_0, f(x_0))$ gives a line
- Definition: $f$ is differentiable at $x_0$ if $\lim\limits_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$ exists

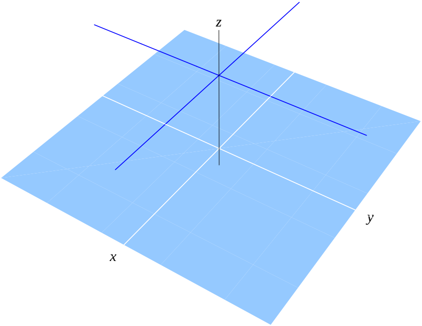
For $f : \mathbb{R}^2 \to \mathbb{R}$

- Idea: $f$ is differentiable at $(x_0, y_0)$ if zooming in on the graph at $(x_0, y_0, f(x_0, y_0))$ gives a plane
- Definition: ???

It is not enough to know that $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ exist.

# Example: a nondifferentiable function

$$f(x,y) = \begin{cases} 1 & \text{if } x = 0 \text{ or } y = 0 \\ 0 & \text{otherwise} \end{cases}$$




Note that $f_x(0,0) = 0$ and $f_y(0,0) = 0$ so the partial derivatives exist for $(0,0)$ *but* zooming in on $(0,0,1)$ does not give a plane.

# A sufficient condition for differentiability

Defining **differentiable** for $f : \mathbb{R}^2 \to \mathbb{R}$ is a bit messy.

Can give a sufficient condition for differentiability:

**Theorem:**
If $(x_0, y_0)$ is a point in the domain of a function $f$ with

(A) $f$ defined for all points in an open disk centered at $(x_0, y_0)$, and

(B) $f_x$ and $f_y$ each continuous for all points in that open disk

then $f$ is differentiable for $(x_0, y_0)$.

# Differentiability as a hypothesis for other results

Differentiability is often a condition needed as a hypothesis.

**Theorem:**
If $f$ is differentiable in an open region containing the point $P$, then

$$\left(\frac{df}{ds}\right)_{P,\hat{u}} = \vec{\nabla} f(P) \cdot \hat{u}.$$