Name ____________________

MATH 302 | Partial Differential Equations | Fall 2010 | Exam #10

Instructions: Do your own work. You may consult your class notes, the course text, and similar resources from prerequisite courses (calculus, linear algebra, and ordinary differential equations). Do not consult other sources. Do not discuss generalities or specifics of the exam with anyone except me.

Turn in a complete and concise write up of your work. Show enough detail so that a peer could follow your work (both computations and reasoning). All plots should be carefully drawn either by hand or printed from technology. If you want to include a visualization that cannot be printed (such as an animation), include it as an attachment in an email with "Math 302 Exam 10" as the subject line.

The exam is due in class on Monday, December 6.

Pick *one* of the following two problems to submit.

1. Consider a rod of length $l = 2$ meters made from material with diffusivity $k = 1.14 \times 10^{-4}$ m^2/s. The rod is thin and the lateral sides of the rod are insulated so heat can effectively only flow along one direction. One end of the rod is insulated and the other end is allowed to convect heat into the surrounding environment, which is at a constant temperature of 0°C. The rod is intially at a uniform temperature of $T_0 = 50$°C. A reasonable model for this situation is given by

$$\begin{aligned} u_t &= k u_{xx} && \text{for } 0 < x < l,\ t > 0 \\ u_x(0,t) &= 0 && \text{for } t > 0 \\ u_x(l,t) &= -\alpha u(l,t) && \text{for } t > 0 \\ u(x,0) &= T_0 && \text{for } x > 0 \end{aligned}$$

with $\alpha = 1.0$ m^{-1}. Determine (at least approximately) how long it takes for the temperature at $x = 0$ to reach 10°C.

2. You want to model how your guitar will sound if you submerge it in a vat of honey. Honey resistance will be important so you decide to include a damping term in your model. A simple model is to assume a damping force that is proportional to velocity. Including a term of this form in the wave equation gives us the PDE

$$u_{tt} = c^2 u_{xx} - \beta u_t.$$

   (a) Set up an initial-boundary value problem for this PDE that models vibrations on a string of length l with both ends held fixed in which the string is displaced from equilibrium and then released from rest.

   (b) Find a solution to the IBVP you set up in (a) for the case $\beta < \dfrac{2\pi c}{l}$. (This condition guarantees underdamping for all modes.)

   (c) Illustrate your solution from (b) using specific parameter values (string length l, wave speed c, and damping coefficient β) and a specific initial displacement of your own choice other than the trivial case of zero initial displacement.