A Sturm-Liouville problem

Consider the Sturm-Liouville problem

$$y''(x) = -\lambda y(x) \qquad \text{for } 0 < x < l$$
$$y(0) = 0$$
$$y(l) + y'(l) = 0$$

1. Use a Raleigh quotient (or energy) argument to bound the set of eigenvalues.
   Hint: Multiply both sides of the ODE by y and then integrate both sides from $x = 0$ to $x = l$.

2. Find a "characteristic equation" for the eigenvalues.

3. Use your equation from 2 to argue that there are countably many eigenvalues λ_n that can be ordered as $\lambda_1 < \lambda_2 < \dots$ with $\lambda_n \to \infty$ as $n \to \infty$.
   Hint: Think graphically.

4. Estimate λ_1, λ_2, and λ_3.

5. Come up with a formula that gives approximate values of λ_n for large values of n.