Notes on *Mathematica*

Mathematica is a general purpose mathematics software system. You can think of it as a powerful graphing calculator. When it originated, *Mathematica* was often to referred to as a *computer algebra system (CAS)*. CAS programs are characterized by allowing for *symbolic* calculations in contrast to *numeric* calculations. The phrase CAS does not fully describe the capabilities of *Mathematica* or other similar programs.

Other similar commercial programs/packages include *Matlab* and *Maple*. *Sage* is an open source system that is being developed as a freely available alternative to commercial products. You can check it out at `sagemath.org`

These systems have different origins but have converged to include roughly the same capabilities. Different points of origin mean each has different relative strengths and weaknesses. I have not carefully evaluated those strengths and weaknesses. I use *Mathematica* primarily because it is the system I learned first and now know best.

Here's some quick notes on getting started:

- basic operation: type input, hit enter or shift-return for output

- two pieces: front end and kernel

- note In[] and Out[] labels added

- can refer to previous output using Out[x] or %x

- % refers to the most recent output

- note cell structure

- can edit, copy and paste

- order of execution is what matters, not order of appearance in the notebook

- multiple notations: keyboard, palettes, Esc

- multiplication with * or space

- role of brackets: square, curly, rounded

- distinguish among = and := and ==

- built-in values and commands use upper case (Pi, E, I, Factor)

- built-in functions have required arguments and optional arguments

- assigments are global and persistant unless you say otherwise

- assignments are not automatically initialized unless you say otherwise

- documentation is available

The accompanying handout lists a variety of inputs for you to try. For each, you should type in the given input and then have *Mathematica* produce the corresponding output. Study the output to see how it relates to the input. Experiment with variations on the input to see what each piece of the input does.

Introduction to Mathematica

The best way to learn about *Mathematica* is to play around with a variety of things the program can do. Below is a list of inputs. One at a time, type in each input and have *Mathematica* process the input by hitting SHIFT+RETURN or the ENTER key on the number pad. Look at the output and try to figure out how it relates to the input you entered. Experiment with changing parts of the input to get different outputs. Note that you can edit an input or copy it and paste it elsewhere.

- **Numbers and arithmetic**

    ```
    Pi

    N[Pi]

    N[Pi, 50]

    2 Pi

    2. Pi

    2 + 3

    2 * 3

    2 × 3

    2 / 3

    2. / 3
    ```

- **Algebra and equations**

    ```
    Factor[x^2 - 5 x + 6]

    a = (x + 1)^2 + (3 x + 2)^3

    Expand[a]

    b = (3 x + 1)^2 / x + 2 / x^2

    Together[b]

    Simplify[b]

    Solve[x^2 + 5 x + 6 == 0, x]

    Solve[Sin[x] + Cos[x] == x, x]

    FindRoot[Sin[x] + Cos[x] == x, {x, 0}]

    FindRoot[Sin[x] + Cos[x] == x, {x, 1}]
    ```

Note: When using FindRoot, it's often best to have a graph at hand. See below for details on plotting in *Mathematica*.

- **Functions**

    ```
    f[x] = x^3

    f[x]

    f[3]

    Clear[f]

    f[x_Real] = x^3

    f[x]

    f[3]

    f[3.]

    f[3 + I]

    Clear[f]
    ```

```
Clear[f]
f[x_] = x^3
f[x]
f[3]
f[3.]
f[3 + i]
f[{3, 2}]
Clear[f]
```

Note: It is good practice to clear any assignments you no longer need.

- **Graphics**

```
Plot[Sin[x], {x, 0, 2 Pi}]
Plot[Sin[x], {x, 0, 2 Pi},
 PlotRange → {{0, 4 Pi}, {-2, 2}},
 PlotStyle → {Dashed, Thick, Red},
 Frame → True,
 Axes → False,
 AspectRatio → 1]
Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}]
plot1 = Plot[Sin[x], {x, 0, 2 Pi}]
plot2 = Plot[Cos[x], {x, 0, 2 Pi}]
Show[plot1, plot2]
Graphics[{Line[{{0, 1}, {1, 1}}], Circle[{0, 0}, 2]}]
Plot3D[Sin[x] Sin[y], {x, 0, 4 Pi}, {y, 0, 4 Pi}]
ContourPlot[Sin[x] Sin[y], {x, 0, 4 Pi}, {y, 0, 4 Pi}]
Manipulate[Plot[a * Sin[x], {x, 0, 2 Pi}], {a, -1, 1}]
Manipulate[Plot[a * Sin[x], {x, 0, 2 Pi}, PlotRange → {-1, 1}], {a, -1, 1}]
Animate[Plot[a * Sin[x], {x, 0, 2 Pi}, PlotRange → {-1, 1}], {a, -1, 1}]
```

- **Calculus**

```
D[x^3, x]
D[x^3, {x, 2}]
f[x_] = x^2
f'[x]
f''[x]
Integrate[x^2, x]
Integrate[x^2, {x, 1, 3}]
NIntegrate[x^2, {x, 1, 3}]
Integrate[x^2, {x, a, b}]
Integrate[Sin[n x], {x, 0, 2 Pi}]
Integrate[Sin[n x], {x, 0, 2 Pi}, Assumptions → n ∈ Integers]
Integrate[Exp[-x^2], x]
Integrate[Exp[-x^3], x]
```

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Linear algebra

```
a = {{1, 3}, {4, 2}}

MatrixForm[a]

a // MatrixForm
b = {{4, -2}, {1, 5}}

a * b

a.b

Det[a]

Tr[a]

Inverse[a]

Transpose[a]

Eigenvalues[a]

Eigenvectors[a]

Eigensystem[a]
```

- **Formatting**

Mathematica has formatting features to produce presentable documents. You can choose different styles for different cells including text cells, display formula cells, and sectioning cells. Section, subsection, and subsubsections cells have styled headers for section titles and producing a cell hierarchy for outlining. This hierarchy can be collapsed or expanded at each level by double-clicking on the corresponding cell bracket.

For practice, reproduce the following cells. You might find it useful to open the "Writing Assistant" palette under the "Palettes" menu.

This is a text cell. You can write formatted inline equations such as $x^2 + y^2 = 1$. You can also write display line formulas such as

$$\cos^2 \theta + \sin^2 \theta = 1.$$ (1)

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