Notes on $L^2(a, b)$

- $L^2(a, b)$ is a vector space

- elements in $L^2(a, b)$ are functions that are *square-integrable* for the domain (a, b)

  - a function is *square-integrable* if the square of the function is integrable

  - that is, a function is *square-integrable* if $\int_a^b (f(x))^2 \, dx$ exists

- addition is pointwise addition: $(f + g)(x) = f(x) + g(x)$ for each x in (a, b)

- scalar multiplication is pointwise multiplication: $(cf)(x) = c\,f(x)$ for each x in (a, b)

- if f is bounded and has at most a finite number of discontinuities for (a, b), then f is in $L^2(a, b)$

- if f is unbounded (i.e., has a vertical asymptote in (a, b)), then need to check convergence of an improper integral

- the L in $L^2(a, b)$ stands for *Lebesgue*

  - in calculus, you learn about *Riemann integrals*

  - *Lebesgue* integrals are a generalization of Riemann integrals

  - with Riemann integrals, improper integrals are handled as a special case

  - with Lebesgue integrals, improper integrals are handled in the same way as proper integrals

  - Lebesgue integration provides a unified way of handling a larger class of functions than does Riemann integration

  - distinction between Riemann integrals and Lebesgue integrals is not important for this course