

Notes and problems on flux

To account for stuff as it flows across a surface, we introduce the idea of *flux*. Flux is a measure whose

- direction gives the direction of flow at each point on the surface and each time
- magnitude gives the *area density for flow rate* at each point on the surface and each time

In general, flux is a vector quantity to account for direction of flow and rate of flow. To begin, we will limit our attention to a special case that does not require a vector description.

To be concrete, consider flow that is parallel to a fixed line and consider only surfaces perpendicular to that line. If we set up a coordinate axis along the line, we can use sign (positive or negative) to account for the two possible directions. Let x be the coordinate for the line with $x > 0$ to the right and $x < 0$ to the left of a chosen origin. Let $\phi(x, t)$ denote the flux at position x and time t with $\phi > 0$ corresponding to flow to the right and $\phi < 0$ corresponding to flow to the left.

The phrase “area density for flow rate” is worth parsing carefully. One way to do this is to consider units. Suppose our measure of the amount of stuff is mass in units of kilograms (kg). A flow rate then has units of kilograms per second (kg/s). A flow rate area density thus has units of kilograms per second per square meter:

$$\frac{\text{kg/s}}{\text{m}^2} = \frac{\text{kg}}{\text{s} \cdot \text{m}^2}.$$

The first form is more informative but the second is commonly used.

As a specific example, consider a flow with a uniform flux of 5 (kg/s)/m^2 . So, for every square meter of the surface, stuff flows through at a rate of 5 kilograms per second. To get flow rate from flux, we multiply by surface area. For our flux magnitude of 5 (kg/s)/m^2 through a surface of area $A = 0.15 \text{ m}^2$, we get

$$\text{flow rate} = (\text{flux})(\text{area}) = \left(5 \frac{\text{kg/s}}{\text{m}^2}\right)(0.15 \text{ m}^2) = 0.75 \frac{\text{kg}}{\text{s}}.$$

So if the flow remains constant in time, then 0.75 kilograms of stuff passes through the surface every second. The fact that this flux and flow rate are positive means that the flow is carrying mass to the right through the surface. (Keep in mind that we are still considering the special case of flow along a line through a surface perpendicular to that line.)

Notes:

- The surfaces we consider in thinking about flux might be real physical surfaces (such as a net through which fluid is flowing) or conceptual surfaces we introduce.
- In general, flux is a vector quantity that can vary from point to point on the surface under consideration. In this case, the total flow is computed as a surface integral over the surface S :

$$\text{flow rate} = \iint_S \vec{\phi} \cdot d\vec{A}.$$

- Those who have done electromagnetism in a physics course will have seen *electric flux*. Given an electric field \vec{E} , electric flux for the surface S is usually defined as $\Phi = \iint_S \vec{E} \cdot d\vec{A}$. This parallels the surface integral in the previous note. In this context, however, nothing is flowing from one place to another. Also note that the use of the word “flux” here is *not* consistent with the way we are using flux above. Our quantity ϕ is a density while the electric flux Φ is a total. The electric field itself plays the role of our flux (as a density).

Problems

1. Electrons flowing in a wire carry charge (measured in Coulombs (C)) with a flux of 0.03 (C/s)/m^2 .
 - (a) What is the rate at which charge flows through a cross section (perpendicular to the wire) of area 0.000004 m^2 ?
 - (b) What is the rate at which charge flows through a circular cross section of radius 0.002 m ?
2. The charge on one electron is $1.6 \times 10^{-19} \text{ C}$.
 - (a) What is the number flux corresponding to a charge flux of 0.03 (C/s)/m^2 ?
 - (b) What is the rate at which electrons flow through a cross section (perpendicular to the wire) of area 0.000004 m^2 ?
3. For fluid flowing in a pipe, we can measure the volume of the fluid and consider the corresponding flux. This would be the area density for the flow rate of volume.
 - (a) Determine the units on flux in this case. Give both a form that reveals the meaning of flux and a completely simplified form.
 - (b) Based on your units analysis in (a), conjecture a relationship between flux and fluid velocity.
 - (c) Develop a geometric argument to support your conjecture in (b) for the simple case in which the fluid velocity is constant in time.