Name ______________________

MATH 280 Multivariate Calculus Spring 2007 Exam #3

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. (a) State the definition for each of the partial derivatives $\dfrac{\partial G}{\partial u}$ and $\dfrac{\partial G}{\partial v}$ for the function $G(u,v)$. (Give a definition, not an interpretation such as rate of change or slope. Note that the definition involves a difference quotient.) (6 points)

   (b) Suppose G is measured in units called *blargs*, u is measured in units called *mehs*, and v is measured in units called *d'ohs*. What are the units of $\dfrac{\partial G}{\partial u}$ and $\dfrac{\partial G}{\partial v}$? (4 points)

2. Compute all first and second partial derivatives for the function $f(x,y) = \sin(xy^2)$. (12 points)

3. We are given the following information for a certain function $f(x,y)$ of two variables:

$$f(1,4) = 3, \qquad \frac{\partial f}{\partial x}(1,4) = -2, \qquad \text{and} \qquad \frac{\partial f}{\partial y}(1,4) = 1.5.$$

   The variables x and y themselves depend on two other variables u and v as given by

$$x(u,v) = 3 + 2u \qquad \text{and} \qquad y(u,v) = uv - 2$$

   Compute $\dfrac{\partial f}{\partial u}$ for $(u,v) = (-1,-6)$. (10 points)

4. Consider the function $f(x,y,z) = xy + yz^2$. For the point $(3,1,2)$, find the direction (as a unit vector) in which the rate of change in f is greatest and find the value of that greatest rate of change. (12 points)

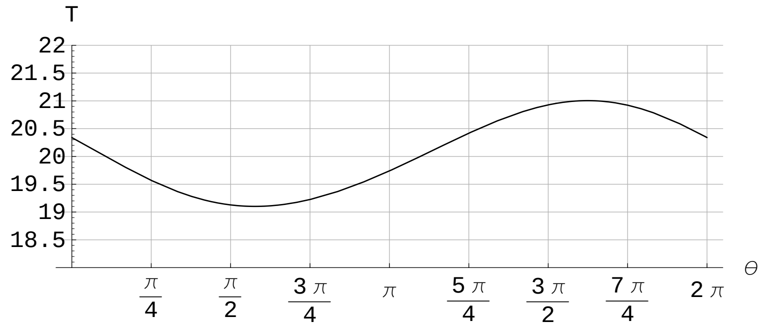
5. For a certain function of two variables, the gradient at $(3-1)$ is given by

$$\vec{\nabla} f(3,-1) = 2.1\,\hat{\imath} - 3.4\,\hat{\jmath}.$$

   Estimate the change in f for a displacement from the point $(3,-1)$ of size 0.05 in the direction of the point $(1,5)$. (12 points)

There is more on the flip side.

6. A cockroach measures the temperature at a point P on the table to be 20.0 °C. The cockroach then measures the temperature at every point on a circle of radius 2.0 cm centered at P. Data from these measurements is given in the plot below as temperature along the circle versus the angle around the circle. Estimate the direction (as an angle around the circle) and magnitude of the temperature gradient vector for the point P. Include units in your magnitude estimate. (10 points)



7. A economist is modeling the utility a consumer assigns to a bundle of two commodities. Let r and s be the amounts of each commodity and suppose the utility is given by

$$U(r,s) = r^{1/4}s^{3/4}.$$

(a) Find the relationship of the (approximate) percentage change in utility to percentage changes in r and s. (9 points)

(b) Use your relation from (a) to give determine the (approximate) percentage change in utility that results from increasing r by 1% and decreasing s by 2%. (3 points)

8. Consider the function $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$.

(a) Show that $(-1,2)$ is a critical point for f. (6 points)

(b) Use the second derivative test to determine if $(-1,2)$ is a local minimizer, a local maximizer, or neither. (6 points)

9. You manufacture two products in quantities q_1 and q_2. You sell each unit of the first product for \$6 and each unit of the second product for \$4. The total cost (in dollars) to produce these quantities is given by

$$C = 0.05q_1^2 + 0.01q_2^2 + 0.02q_1q_2.$$

Find the values of q_1 and q_2 to maximize your profit. (10 points)