Name ______________________________

MATH 280 | Multivariate Calculus | Spring 2007 | Exam #1

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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1. (a) Draw a figure illustrating two generic vectors $\vec{u}$ and $\vec{v}$ and the vector $4\vec{v} - 2\vec{u}$. (5 points)

   (b) Compute the components of $4\vec{v} - 2\vec{u}$ for $\vec{u} = 5\,\hat{\imath} + 2\,\hat{\jmath} - 6\,\hat{k}$ and $\vec{v} = 3\,\hat{\imath} - \,\hat{\jmath} + 4\,\hat{k}$. (5 points)

2. A submarine moving 15.7 miles due north enters an ocean current that is moving 3.6 miles per hour west and 2.1 miles per hour down. The new velocity of the submarine is the sum of its original velocity and the ocean current velocity. What is the speed and what is the direction of the submarine after it enters the current? Give the direction as a unit vector in terms of $\hat{\imath}$ pointing due east, $\hat{\jmath}$ pointing due north, and $\hat{k}$ pointing up. (10 points)

3. For each of the following, give both a geometric definition and a component expression:

   (a) the dot product of $\vec{u}$ and $\vec{v}$ (6 points)

   (b) the cross product of $\vec{u}$ and $\vec{v}$ (6 points)

4. For each of the following, determine if the given expression is a scalar, a vector, or undefined. If undefined, explain why. (2 points each)

   (a) $(\vec{u} \times \vec{v}\,) \cdot \vec{w}$

   (b) $(\vec{u} \cdot \vec{v}\,) \times \vec{w}$

   (c) $\left[(\vec{p} \cdot \vec{q}\,)\,\vec{r}\,\right] \times \vec{s}$

   (d) $(\vec{p} \times \vec{q}\,) \cdot (\vec{r} \times \vec{s}\,)$

5. Find the angle between the vectors $2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $-5\hat{\imath} + 2\hat{\jmath} + \hat{k}$. (10 points)

6. Find a vector that is perpendicular to each of the vectors $2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $-5\hat{\imath} + 2\hat{\jmath} + \hat{k}$. (8 points)

There is more on the flip side.

7. Consider a plane that contains the point P_0 and is perpendicular to the vector $\vec{n}$.

   (a) Explain how we know the point P is on the plane if $\vec{n} \cdot \overrightarrow{P_0P} = 0$. You can use words and pictures for this. (5 points)

   (b) Show how to go from the vector equation $\vec{n} \cdot \overrightarrow{P_0P} = 0$ to a coordinate equation of the form $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. (5 points)

8. Find a coordinate equation for the plane that contains the points $Q(6, 1, 3)$, $R(4, 2, 1)$, and $S(6, -5, 2)$. (10 points)

9. (a) To compute the distance d between a point P and a plane, we can use the formula

$$d = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|}$$

   where Q is any point on the plane and $\vec{n}$ is a normal vector for the plane. Give an argument to justify this formula. You can use words and pictures for this. (4 points)

   (b) Compute the distance between the point $P(6, 1, 2)$ and the plane given by the equation $2x + 4y - 3z = 12$. (8 points)

10. Do either one of the following two problems. Circle the number of the problem you are submitting. (10 points)

   (A) For a cube, there are two types of diagonal to consider. A *body diagonal* goes from one corner to the opposite corner through the body of the cube. A *face diagonal* goes from one corner to the opposite corner on the same face. Use vectors to find the angle between a body diagonal and a face diagonal that meet at a corner.

   (B) The figure below shows a parallelogram $ABCD$ and the midpoint P of the diagonal BD.

      i. Express $\overrightarrow{AP}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

      ii. Prove that P is also the midpoint of the diagonal AC.