Total from length density

1. Charge is distributed on a line segment of length L so that the length charge density is proportional to the square of the distance from one end, reaching a maximum density of λ_0 at the other end. Compute the total charge on the segment.

   Answer: $Q = \frac{1}{3}\lambda_0 L$

2. Charge is spread on a circle of radius R so that the length charge density varies around the circle. (Note that circle here means the curve as opposed to a disk.) Let λ be the charge density measured in Coulombs per meter (C/m). Let θ measure the angle on the circle from a fixed reference ray (conventionally taken to be the positive x-axis). So, the charge density λ varies with angle θ.

   (a) Construct a definite integral to compute the total charge on the circle. Note: Since we do not yet have a specific density function, we cannot yet evaluate this integral.

   (b) Compute the total charge if $\lambda(\theta) = \lambda_0(1 + \cos\theta)$ where λ_0 is a positive constant.

   (c) Compute the total charge if $\lambda(\theta) = \lambda_0 \cos^2\theta$ where λ_0 is a positive constant.

   (d) Get a numerical value for the total charge in (c) using the values $R = 0.25$ m and $\lambda_0 = 1.6 \times 10^{-3}$ C/m.

   Answer: (b) $Q = 2\pi R\lambda_0$ (c) $Q = \pi R\lambda_0$