Integration over a surface problems

1. Compute the surface area of a sphere of radius R

Answer: $A = 4\pi R^2$

2. Compute the surface area of the lateral side of a right circular cone of height H and radius R.

Answer: $A = \pi R\sqrt{R^2 + H^2}$

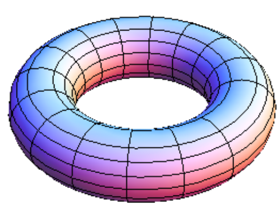
3. A *torus* is the doughnut-shaped surface formed by bending and gluing a right circular cylinder section so that the central axis forms a circle. (You can also think about a torus as generated by revolving circle around a fixed axis that does not intersect the circle.) Let R be the radius of the cylinder and B be the height of the cylinder. (Equivalently, R is the radius of the circle and B is the distance from the circle center to the rotation axis.) The cartesian coordinates of points on a torus can be described by the equations

$$x = (B + R\sin\phi)\cos\theta, \qquad y = (B + R\sin\phi)\sin\theta, \qquad \text{and} \qquad z = R\cos\phi.$$

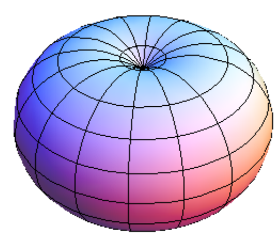
for $0 \le \phi \le 2\pi$ and $0 \le \theta \le 2\pi$. Note that we need $B > R$ to have a true torus. If $B < R$, the surface is a "sphere with dimples at the north and south pole". If $B = 0$, then these formulas describe a sphere of radius R (covered *twice* since $0 \le \phi \le 2\pi$).

Compute the surface area of a torus with dimensions B and R with $B > R$.

Answer: $A = 4\pi^2 BR$



$B = 3$ and $R = 1$



$B = 2$ and $R = 3$

4. Charge is distributed on a hemisphere of radius R so that the area charge density is proportional to the distance from the equatorial plane. Compute the total charge in terms of R and the maximum density σ_0.