# Integration over a curve problems

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- If you cannot easily evaluate a given integral exactly (because finding an antiderivative is difficult or not possible analytically), you should try getting a good numerical approximation using technology such as the `fnInt` feature on a TI-83/84.
- After computing a length or total quantity, you should check if your result is reasonable by finding "easy-to-compute" comparisons.

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1. Compute the length of the segment of the parabola $y = x^2$ for $-1 \leq x \leq 1$.

   *Answer:* $L = \int_{-1}^{1} \sqrt{1 + 4x^2}\, dx \approx 2.958$

2. Compute the length of a circle of radius $R$.

   *Answer:* $L = 2\pi R$

3. Compute the length of the segment of the cubic curve $y = x^3$ for $-1 \leq x \leq 1$.

   *Answer:* $L = \int_{-1}^{1} \sqrt{1 + 9x^4}\, dx \approx 3.096$

4. Compute the length of the segment of the sine curve $y = \sin(x)$ for $0 \leq x \leq 2\pi$.

   *Answer:* $L = \int_{0}^{2\pi} \sqrt{1 + \cos^2 x}\, dx \approx 7.640$

5. A curve in the plane is described parametrically by $x = t^2$, $y = t^3$ for $0 \leq t \leq 2$. (You can think of this as describing the path of an object moving in time with $(x, y) = (t^2, t^3)$ being the position of the object for time $t$.) Compute the length of the curve.

   *Answer:* $L = \dfrac{8(10\sqrt{10} - 1)}{27}$

6. Compute the length of the helix that wraps 5 times around the lateral side of a right circular cylinder of radius $R$ and height $H$ with a constant pitch (so each wrap rises the same distance up the cylinder).

   *Answer:* $L = \sqrt{(10\pi R)^2 + h^2}$

*There is more on the flip side.*

7. Compute the length of the helix that wraps $n$ times around the lateral side of a right circular cone of radius $R$ and height $H$ with a constant pitch (so each wrap rises the same distance up the cone). The helix starts at the vertex of the cone.

$$\textit{Answer: } L = \frac{1}{2\pi n}\int_0^{2\pi n} \sqrt{H^2 + R^2 + R^2\theta^2}\, d\theta$$

8. A curve in space is described parametrically by $x = t$, $y = t^2$, and $z = t^3$ for $0 \le t \le 2$. (You can think of this as describing the path of an object moving in time with $(x, y, z) = (t, t^2, t^3)$ being the position of the object for time $t$.) Compute the length of the curve.

$$\textit{Answer: } L = \int_0^2 \sqrt{1 + 4t^2 + 9t^4}\, dt \approx 9.571$$

9. Charge is distributed on a semicircle of radius $R$ so that the length charge density is proportional to the distance from the diameter that contains the two ends of the semicircle. Let $\lambda_0$ be the maximum charge density. Compute the total charge $Q$.

$$\textit{Answer: } Q = 2R\lambda_0$$

10. A piece of wire has the shape of the parabola $y = \frac{b}{a^2}x^2$ for $-a \le x \le a$ where $a$ and $b$ are positive constants (each carrying units of length). The wire has a non-uniform composition so that the length mass density is proportional to the square root of the distance from the $x$-axis reaching a maximum density $\lambda_0$. Compute the total mass of the wire.