

Total from area density

1. Charge is distributed on a flat rectangular region of dimensions L by W so that the charge density is proportional to the distance from one corner, reaching a maximum of σ_0 at the far corner. Set up an iterated integral to compute the total charge Q . You do not need to evaluate this integral. As an optional challenge, you can try to evaluate the iterated integral.

$$\text{Answer: } Q = \frac{\sigma_0}{\sqrt{L^2 + W^2}} \int_0^W \int_0^L \sqrt{x^2 + y^2} \, dx dy$$

2. Charge is distributed on an isosceles triangle of height H and base length B so that the charge density is proportional to the distance from the base, reaching a maximum of σ_0 at the vertex opposite the base. Compute the total charge Q .

$$\text{Answer: } Q = \frac{1}{6} BH \sigma_0$$

3. Charge is distributed on the surface of an open right circular cylinder of radius R and height H so that the area charge density is proportional to the distance from one end of the cylinder, reaching a maximum of σ_0 at the other end. Compute the total charge Q .

Note: This situation involves an area density. Depending on how you approach the problem, you might end up constructing and evaluating an integral in just one variable.

$$\text{Answer: } Q = \pi RH \sigma_0$$