Applied optimization problems

1. The United States Postal Service web site states "The maximum size for most mailpieces is 108 inches in combined length and girth." Find the maximum volume of a rectangular package allowed under this condition.
   Note: Length is the longest of the three dimensions. Girth is the perimeter of the rectangle with the two shorter dimensions.

2. Consider the problem of designing a box to meet certain specifications at minimum cost. The design specifications call for a rectangular box of total volume V (in cm^3). The box is to be constructed with material for the top that costs a (in dollars per cm^2), material for the bottom that costs b (in dollars per cm^2), and material for the sides that costs c (in dollars per cm^2). Find the dimensions of the box that meets the specifications at minimum cost.
   Note: You should expect the minimizing dimensions to be in terms of the parameters in the problem (V, a, b, and c).

3. You own a manufacturing company that produces two versions of a video game console, a standard version and an advanced version. Basic economic theory tells us that the price of each will decrease as the available quantities increase. Let q_1 and q_2 be the available quantities of the standard and advanced versions, respectively. Let p_1 and p_2 be their selling prices in dollars. Your marketing department has produced the following model for the relationships among these variables:

   $$p_1 = 400 - 0.1q_1 - 0.04q_2 \qquad \text{and} \qquad p_2 = 500 - 0.02q_1 - 0.6q_2$$

   The cost for you to manufacture and distribute each unit of the standard version is \$50 while the cost for the advanced unit is \$65. How many units of each should you make available in order the maximize your profit?

4. Consider studying consumer utility for a bundle of goods consisting of pizza, coffee, and textbooks. Let p, c, and t be the amounts of pizza, coffee, and textbooks in the bundle. Assume the utility for this bundle is given by

   $$U(p, c, t) = p^{1/3}c^{1/2}t^{1/6}.$$

   Suppose the unit costs of pizza, coffee, and textbooks are \$10, \$2, and \$80, respectively. A consumer has a total of \$1000 available. Find the combination of pizza, coffee, and textbooks that maximizes the utility for this consumer.
   Note: Don't worry if your results are not whole numbers.