

**Instructions:** Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

*For this exam, you should not use symbolic features on your calculator if they are available on your model.*

1. Below on the left are four ideas (in summary) about *definite integral*. On the right are four descriptions of these ideas. Write a brief paragraph that matches each idea on the left with a description on the right *and* provides some connections among these ideas. You should write from the perspective on these ideas we have used in this course. (10 points)

(A)  $\int_a^b f(x) dx = \lim_{\Delta x \rightarrow 0} \sum_{k=1}^n f(c_k) \Delta x$  (i) conclusion of the Second Fundamental Theorem

(B)  $\int_a^b f(x) dx = F(b) - F(a)$  (ii) an application of definite integral

(C) “summing infinitely many infinitely small contributions to a total” (iii) precise definition of definite integral

(D) “area under a curve” (iv) informal idea of definite integral

2. Evaluate the definite integral  $\int_0^1 x \cos(\pi x^2) dx$ . (10 points)

3. Evaluate *one* of the two following indefinite integrals. Circle the number of the problem you submit. (8 points)

(A)  $\int x^2 \ln x dx$

(B)  $\int \frac{1}{x^2 + 5x + 6} dx$

4. Compute the derivative of  $g(x) = \int_x^3 \frac{\sin y}{y} dy$ . (6 points)

5. Compute an approximation of  $\int_0^1 \cos(x^2) dx$  with error less than 0.01. *For this problem, you may not use built-in or programmed integration features on your calculator.* (10 points)

*There is more on the flip side.*

6. A particular glacier is receding (that is, getting shorter in length) at a rate predicted to be  $r(t) = 3 - 2e^{-0.4t}$  meters per year. Compute the total predicted recession for the period  $t = 0$  to  $t = 10$  years. (10 points)

7. Compute the volume of the solid generated by revolving the region bounded between the graphs of  $f(x) = \sqrt{x}$  and  $g(x) = x^2$  around the  $y$ -axis. (10 points)

8. Do *one* of the following two problems. Circle the number of the problem you submit. (10 points)

(A) A rectangular piece of cloth of height  $H$  and width  $W$  is soaked in dye and then hung vertically to dry. As the cloth dries, the dye flows down so that more ends up at the bottom than at the top. The dried dye has a mass density given by  $\sigma(h) = \sigma_0(1 - h^2/H^2)$  where  $h$  is the distance from the top of the cloth. Compute the total mass of dye in the cloth.

(B) A conservation biologist develops a mathematical model for the population  $y$  of a threatened species as a function of time  $t$ . The model is given by the differential equation

$$\frac{dy}{dt} = -kty \quad \text{with} \quad y(0) = y_0.$$

where  $k$  is a positive constant. Find the solution to this differential equation (leaving  $k$  and  $y_0$  as parameters) and then use your solution to find the predicted population for  $t = 10$  using the values  $k = 0.01$  and  $y_0 = 1000$ .

9. Determine whether the series  $\sum_{k=0}^{\infty} \frac{1000^k}{k!}$  converges or diverges. (8 points)

10. Use power series representations to prove Euler's formula  $e^{i\theta} = \cos \theta + i \sin \theta$ . (8 points)

11. Consider the function  $f$  defined on the interval  $[0, 1]$  by  $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{x} & \text{if } 0 < x \leq 1. \end{cases}$

(a) Set up a Riemann sum for  $f$  on  $[0, 1]$  using  $n$  subintervals and right endpoints as inputs. (5 points)

(b) Use your knowledge of series to argue that the limit as  $n \rightarrow \infty$  of the Riemann sum from (a) diverges. (5 points)