Project #3

Background: A hydrogen atom consists of one proton and one electron. A *free*
hydrogen atom is one that experiences no external forces. In a free hydrogen atom,
the electron can be in one of infinitely many discrete states. These states are labeled
by three integers, usually denoted n, l, and m. For each state, there is an *electron
location probability density* that gives the probability volume density for the location
of the electron as a function of position (measured with respect to the proton).

The $n = 3$, $l = 2$, $m = 0$ state of a free hydrogen atom has an electron probability
volume density given by

$$\delta(\rho, \phi, \theta) = \frac{1}{39366\pi}\rho^4 e^{-2\rho/3}(3\cos^2\phi - 1)^2$$

where (ρ, ϕ, θ) are spherical coordinates as we use them in class. The origin of the
coordinate system is the location of the proton. The radial coordinate ρ is measured
in units of *Bohr radii* where the Bohr radius is equal to about 5.3×10^{-11} meters.
(So, for example, $\rho = 2$ means a radial distance of 2 Bohr radii.)

Project: For this project, your objective is to get insight on this specific probablility
density function. Below are some calculations and questions you could pursue in order
to get some insight. (Note that a result from the first suggestion can be used for the
other two suggestions). You are freee to augment these or to take a different approach.

- Compute an expression for the probability of finding the electron between $\rho = a$
  and $\rho = b$ for a hydrogen atom in this state.

- Compute the probability of finding the electron in each of the unit intervals
  $\rho = k$ to $\rho = k + 1$ for k between 0 and 25. In which of these intervals is the
  electron most likely to be found?

- Compute the probability of finding the electron anywhere in space. Does this
  result make sense?

Refer to the earlier handout on general project requirements for details on expec-
tations. One way to approach your writing is to think of authoring an example in a
textbook. Your report should be self-contained so that a reader with a background
in multivariate calculus can follow your reasoning and results. As part of this, you
should introduce the context by either paraphrasing the background given above or
developing your own introduction.

Your project is due on Monday, December 6.