Notes on $L^2(a,b)$

- $L^2(a,b)$ is a vector space
- elements in $L^2(a,b)$ are functions that are *square-integrable* for the domain (a,b)
  - a function is *square-integrable* if the square of the function is integrable
  - that is, a function is *square-integrable* if $\int_a^b (f(x))^2\,dx$ exists
- addition is pointwise addition: $(f+g)(x) = f(x) + g(x)$ for each x in (a,b)
- scalar multiplication is pointwise multiplication: $(cf)(x) = c\,f(x)$ for each x in (a,b)
- if f is bounded and has at most a finite number of discontinuities for (a,b), then f is in $L^2(a,b)$
- if f is unbounded (i.e., has a vertical asymptote in (a,b)), then need to check convergence of an improper integral
- the L in $L^2(a,b)$ stands for *Lebesgue*
  - in calculus, you learn about *Riemann integrals*
  - *Lebesgue* integrals are a generalization of Riemann integrals
  - with Riemann integrals, improper integrals are handled as a special case
  - with Lebesgue integrals, improper integrals are handled in the same way as proper integrals
  - Lebesgue integration provides a unified way of handling a larger class of functions than does Riemann integration
  - distinction between Riemann integrals and Lebesgue integrals is not important for this course