Name ______________________________

MATH 180C Calculus and Analytic Geometry I Spring 2009 Exam #2

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

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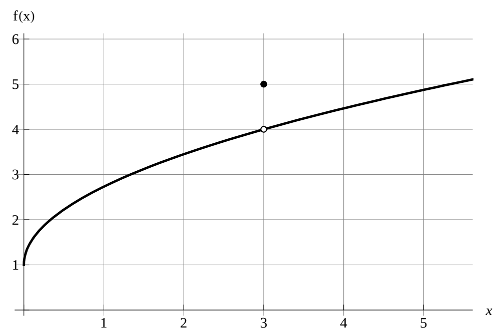
1. (a) State a definition, equivalent to that used in class or the text, for
   $f(x)$ is continuous at $x = a$. (4 points)

   (b) Use the definition from (a) to explain why the function shown in the plot below is *not* continuous at $x = 3$. (4 points)



2. For each of the following, determine if there is a value for A that can be chosen to make the function f continuous at $x = 2$. If so, give that value of A. If not, explain why no value for A works to make f continuous at $x = 2$. (4 points each)

   (a) $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ A & \text{if } x = 2 \\ 6 - x & \text{if } 2 < x \end{cases}$ (b) $f(x) = \begin{cases} x^3 & \text{if } x < 2 \\ A & \text{if } x = 2 \\ x^2 & \text{if } 2 < x \end{cases}$

3. (a) State a definition, equivalent to that used in class or the text, for
   the derivative f' of a function f. (5 points)

   (b) Use the definition from (a) to compute the derivative of the function $f(x) = x^2 + 4$. (5 points)

4. Determine if the function $f(x) = \begin{cases} 5 - x^2 & \text{if } x < 0 \\ 3x + 5 & \text{if } x \geq 0 \end{cases}$ is differentiable at $x = 0$. Explain or indicate how you reach your conclusion. (6 points)

5. Find all values of x for which the graph of the function $f(x) = x^3 - 21x + 16$ has a horizontal tangent line. (8 points)

There is more on the flip side.

6. For each of the following, compute the derivative of the given function. Include reasonable simplification. (7 points each)

   (a) $f(x) = 4x^3 - 3x^2 + 17x - 21$

   (b) $g(x) = (3x^2 - 4x)^6$

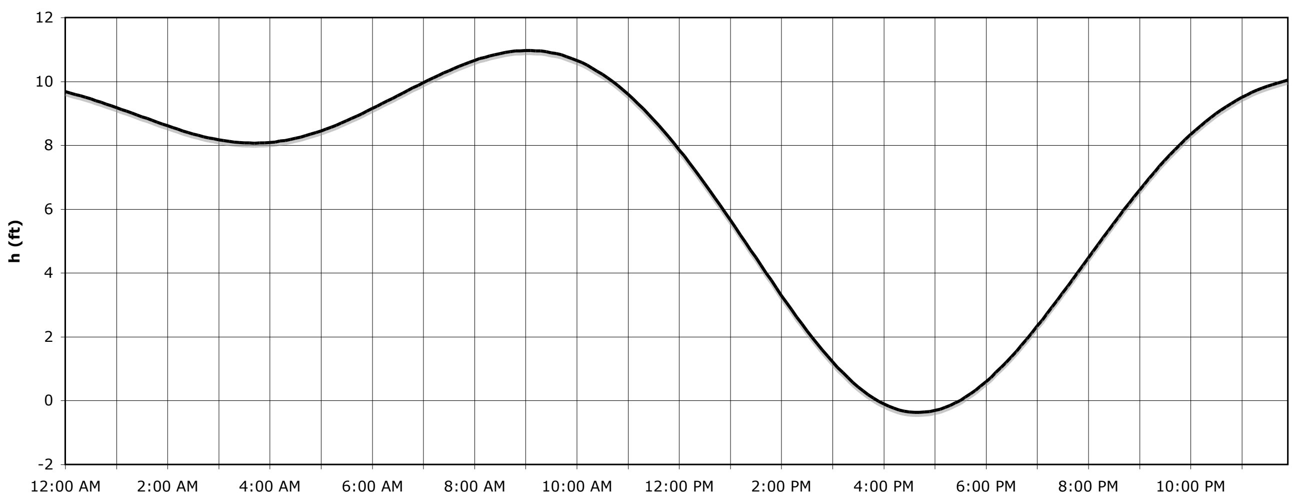
   (c) $y = x^2 e^{5x}$

   (d) $g(t) = \dfrac{t^2+1}{3t+5}$

   (e) $z = 5x^2\cos(2x)$

   (f) $y = \tan(\sqrt{x})$

7. Tides are quantified by measuring the height h of the water surface with respect to a fixed reference level. The graph below shows h (measured in feet) plotted as a function of time t (measured in hours) for the Puget Sound at Tacoma on March 4, 2009.



   (a) Estimate the time at which the tide is rising the fastest. (2 points)

   (b) Estimate the rate at which h is changing with respect to t for the time you found in (a). (3 points)

   (c) Estimate the time at which the tide is lowest. (2 points)

   (d) What is the value of the derivative $h'(t)$ for the time you found in (c)? (3 points)

8. An object is bouncing up and down on the end of a spring. The object's position is given by $y = A\cos(bt)$ where

   - y is measured in centimeters (cm) in reference to the equilibrium position at $y = 0$,
   - t is measured in seconds (s),
   - A is a constant with value $A = 3$ cm, and
   - b is a constant with value $b = 0.2\ \mathrm{s}^{-1}$.

   (a) Compute the velocity of the object for $t = 4$ s. (4 points)

   (b) Compute the acceleration of the object for $t = 4$ s. (4 points)

   Note: Include units in your numerical calculations.