

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

For problems requiring a conclusion about convergence or divergence, provide an argument or evidence to support your conclusion.

Give the result of each indefinite integral in terms of the original variable.

1. State the definition of *the series* $\sum_k^{\infty} a_k$ *converges to the sum* S . (4 points)

2. For each of the following, evaluate the given indefinite or definite integral. *You may not use a calculator for this problem.* (8 points each)

(a) $\int x^2 \ln x \, dx$ (b) $\int_4^8 \frac{x}{x^2 - 5x + 6} \, dx$ (c) $\int \frac{e^x}{1 + e^{2x}} \, dx$

3. Do any two of the following three problems. For each, evaluate the given indefinite or definite integral. *You may not use a calculator for this problem. Circle the letters of the two problems you are submitting.* (8 points each)

(a) $\int \sin^3 x \cos^2 x \, dx$ (b) $\int \frac{\sqrt{1-x^2}}{x} \, dx$ (c) $\int e^{2x} \sin x \, dx$

4. For each of the following, determine if the given improper integral converges or diverges. If it converges, find the value. (8 points each)

(a) $\int_0^{\infty} \frac{x}{x^2 + 1} \, dx$ (b) $\int_{-2}^{-1} \frac{1}{(x+1)^{1/3}} \, dx$

5. For each of the following, determine if the given sequence converges or diverges. If it converges, find the limit. (8 points each)

(a) $a_n = \frac{3n^4 + 5n^2 + 2}{7n^4 + n^3}$ (b) $a_n = \frac{n + \sin n}{n^2}$

6. For each of the following, determine if the given series converges or diverges. If the series is a convergent geometric series, find the sum. (8 points each)

(a) $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ (b) $\sum_{k=1}^{\infty} \frac{5}{4^k}$

7. Water is flowing into a tank at the rate $f(t) = \frac{500}{(t+5)^3}$ in gallons per minute. The tank is empty at $t = 0$. The tank can hold 12 gallons. Will the tank ever overflow?

(8 points)