Name ______________________________

MATH 122B Calculus and Analytic Geometry II Fall 2005 Exam #2

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

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1. For each of the following, evaluate the given indefinite or definite integral. *For this problem, you may not use the symbolic computing features available on a calculator such as the TI-89.* (7 points each)

   (a) $\int x^2(6x^3-5)^{1/3}\,dx$ (b) $\int_0^{\pi/4} \cos(2x)\,dx$

2. Consider the problem of approximating $\int_1^3 \frac{\sin x}{x}\,dx$ using midpoint rectangles.

   (a) Find the minimum number of subintervals needed to get the error less than 0.005. Hint: For $f(x) = \frac{\sin x}{x}$, it is a fact that $|f''(x)| \leq 0.25$ for all x in $[1,3]$. (10 points)

   (b) Compute the midpoint approximation using the number of subintervals you find in (a). Show enough detail to make clear how you are doing this calculation. You don't need to write down every number involved in the calculation. (10 points)

3. Consider the region bounded between the curves $y = 4 - 3x^2$ and $y = x^4$.

   (a) Sketch this region and find any relevant points of intersection. (6 points)

   (b) Set up an integral or integrals to compute the area of this region. *You do not need to evaluate the integral or integrals.* (9 points)

   (c) Now consider generating a solid by revolving this region around the line $y = 5$. Set up an integral or integrals to compute the volume of this solid. *You do not need to evaluate the integral or integrals.* (9 points)

4. Set up an integral or integrals to compute the volume of the following solid: The base of the solid is the region inside the circle $x^2 + y^2 = 16$. Each cross-section perpendicular to the x-axis is an equilateral triangle. *You do not need to evaluate the integral or integrals.*

   Hint: The area of an equilateral triangle of side length l is $A = \frac{\sqrt{3}}{4}l^2$. (10 points)

There is more on the flip side.

5. Set up an integral or integrals to compute the area inside one "petal" of the polar curve $r = \sin(5\theta)$. *You do not need to evaluate the integral or integrals.* (10 points)

6. The density of ocean water increases with depth. Let x be depth (in meters) and $\rho(x)$ be the density (in kg/m^3) at depth x.

   (a) Construct a definite integral to compute the total mass of water in a rectangular column of height H and square base of side length L with the top of the column at sea level. (8 points)

   (b) For depths up to about 500 m, the density increases almost linearly with depth so we can use $\rho(x) = \rho_0 + \alpha x$ where ρ_0 and α are positive constants. Compute the total mass in the column using this density function. (4 point)

   (c) Compute a numerical value for your result from (b) using $\rho_0 = 1025\ \text{kg/m}^3$, $\alpha = 2.1\ (\text{kg/m}^3)/\text{m}$, $H = 20$ m and $L = 2$ m. (2 points)

7. *Do either one of the following two problems. Circle the problem number for the one you are submitting. I will only evaluate work submitted for one problem.* (8 points)

   (a) State the Mean Value Theorem for Integrals. Include hypotheses and conclusion.

   (b) The length of the graph of a function $f(x)$ for the interval $[a, b]$ is given by

   $$s = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\, dx.$$

   Briefly show how this integral is constructed.

Error bound formulas

left/right endpoint: $B_n = \frac{1}{2}\frac{(b-a)^2}{n}M_1$ trapezoid: $B_n = \frac{1}{12}\frac{(b-a)^3}{n^2}M_2$

midpoint: $B_n = \frac{1}{24}\frac{(b-a)^3}{n^2}M_2$ Simpson's: $B_n = \frac{1}{2880}\frac{(b-a)^5}{n^4}M_4$